THE IMPACT OF EQUIPMENT MISPLACEMENT ON PATIENT SERVICE LEVELS IN HOSPITALS

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ABSTRACT

In this paper, we address the problem of equipment misplacement in a hospital and its impact on patient service levels. We develop an analytical framework that can be used to determine the optimal equipment stocking levels for a hospital ward as well as the best directed search policy for recovering any misplaced equipment. We then conduct a numerical study to show how to determine these optimal stocking levels and search policies in a practical setting. Finally, we discuss the implications of our results for determining the suitability of Real Time Locating System (RTLS) implementations in a hospital setting.

Keywords: RFID, RTLS, inventory control, queueing, hospital.

INTRODUCTION

Hospitals need specific equipment to provide quality health care to patients. Examples of such equipment are infusion pumps, bandages, and surgical tools; but also wheelchairs, beds, and gurneys. Since resources in a hospital are limited, patients often have to wait for a specific piece of equipment to become available for their treatment.

Complicating the equipment availability picture is the fact that equipment is frequently misplaced and cannot be found by staff in a reasonable amount of time. Every year, hospitals across the globe spend millions of dollars on searching for misplaced equipment [12]. This represents not only a waste of money but also a waste of caregiving time: according to [10], nurses spent an average of 30 minutes per shift looking for equipment. Thus, when equipment is misplaced, it is not available to satisfy patient demand until it is found, which leads to decreased service level and negatively impacts overall system performance.

The general misplacement phenomenon and potential radio-frequency identification (RFID) or RTLS solutions have been studied extensively in the inventory control literature; for example, see: [3], [4], [8], and [14]. This literature, however, focuses on inventory parts in a retail or distribution environment, not on equipment used in a healthcare setting. The hospital situation is particularly conducive to equipment misplacement due to the stressful environment, the large number of medical personnel that have access to the equipment, and the fact that patients in a hospital are often in a constant state of motion between different facility sections such as radiology, laboratory, pre-/post-op et cetera [1].

One technological attempt to solve the equipment misplacement problem is through an RTLS, which can be installed to continuously track the physical location of equipment. In a typical RTLS, RFID tags are attached to objects and the tags transmit wireless signals to a reader which determines their locations. We refer the reader to [2] as well as [6] for additional discussion of the use of these systems in different application areas.

In this paper, we study the impact of equipment misplacement on patient service levels and provide an analytical framework that can be used to determine the optimal equipment stocking levels for a hospital ward as well as the best directed search policy for recovering any misplaced equipment. Our framework is based on a Markov chain representation of the misplacement process and a queuing theory based modeling approach to patient service levels and optimal stocking levels. A different Markov chain based model has been proposed by [11] to study misplacement in hospitals; however, their work neither discusses optimum stocking levels nor the impact on patient service levels.

MODEL DESCRIPTION

We consider a hospital ward where various equipment such as infusion pumps etc. is stocked for patient usage. We assume that periodically, but randomly, equipment is misplaced; and that misplaced equipment may be found (accidentally, not as part of a directed search effort) by staff. In addition, the ward has a directed search policy similar to a cycle count in place such that whenever less than a certain minimum number of equipment is accounted for, a dedicated search is launched. We assume that this directed search will, after costly effort, locate all of the misplaced equipment.

The Equipment Misplacement Model

We model this system as a Markov chain, where the state of the system is determined by the number of equipment that can be accounted for (that is, where the location is known). Let N be the overall number of equipment that the hospital ward owns. If the hospital has a directed search policy in place such that a search will commence once K or fewer pieces of equipment are accounted for (that is, when N-K pieces are missing), then the system state is an element of the set $\{K,K+1,...,N\}$. Let ω denote the probability that one piece of equipment is misplaced during the time period of one hour, and let σ denote the probability that one piece of equipment is accidentally found by staff, also during one hour. We assume that the probabilities with which a piece of equipment is misplaced or accidentally found are independent of the number of equipment that are on hand. Further, we define ψ to be the probability with which the directed search will locate all misplaced equipment within one hour. Then, the steady-state equations for this Markov chain equipment misplacement model are as follows:

$$\begin{aligned} \pi_N &= \sigma\pi_{N-1} + \psi\pi_K + (1-\omega)\pi_N \\ \pi_i &= \sigma\pi_{i-1} + \omega\pi_{i+1} + (1-\sigma-\omega)\pi_i, \text{ for } K<i<N \\ \pi_K &= \omega\pi_{K+1} + (1-\psi)\pi_K \\ \textstyle\sum^N_{i=K} \pi_i &= 1 \end{aligned} \quad (1)$$

where π_i is the steady state probability of i equipment accounted for in the inventory system.

It is for the hospital management to decide what directed search policy (that is, which threshold K to choose) they want to implement by considering the tradeoff between the cost of search operations and the loss in patient service level due to misplaced equipment. In order to estimate the costs required to search we need to know how often a search takes place during a certain time horizon, given a choice of K.

Proposition 1. Let R be the fundamental matrix of size (N-K)×(N-K) of the absorbing Markov chain specified above for ψ=0. Then the expected time in hours between two consecutive directed searches is given by

$$T = (R^{-1} u)^T e_1 \quad (2)$$

where u is the column vector of ones of size N-K and e_1 is the unit vector that represents the first column of the identity matrix of size N-K.

Proof. Note that by setting ψ=0, the state K becomes an absorbing state. A standard approach for determining the number of steps until reaching an absorbing state is to calculate the fundamental matrix $R=(I-Q)^{-1}$, where I is the identity matrix and Q is the transition sub-matrix of transient states in the canonical form of the transition matrix of the absorbing Markov chain. Then, the first element of $R^{-1}u$ gives the expected number of steps to reach the absorbing state from state N (see, for example, [7]). Since the transition probabilities are defined in units of "per hour", the number of hours between consecutive directed searches follows immediately as given in the proposition.

Given the expected time between successive directed searches, the expected cost of directed search operations performed over a given time horizon of H hours can now be calculated as

$$DSC = cH\,(\psi T)^{-1} \qquad (3)$$

where c is the unit cost of search per hour, which may be estimated from known labor costs.
The Markov chain model described above provides a framework for the equipment misplacement problem. With this model in place, we now examine how a hospital should optimally decide on (a) how many instances of a particular equipment type to own, and (b) at what point to start a search. Thus, these research questions essentially require finding the optimal values of the parameters N and K in the Markov model. In order to answer these questions, we need to define the concept of patient service level.

The Patient Service Level
Consider a type of equipment, such as an infusion pump, that needs to be ready for usage by patients. For simplicity, we assume that one piece of equipment is used per patient. Then, equipment usage can be modeled as a queue in which the N pieces of equipment that are accounted for in the inventory system are the servers, and the patients enter a single line and wait until a piece of equipment becomes available. Note the distinction between a piece of equipment that is accounted for and one that is available. "Accounted for" describes equipment that is known to the inventory system (that is, has not been misplaced). "Available" refers to equipment that is accounted for and that is not in use by a patient. Thus, at a given moment in time, a hospital ward may have 20 infusion pumps that are accounted for, 5 that have been misplaced, and 0 infusion pumps that are available, because all 20 known pumps are in use. We point out that these definitions make it easy to investigate the impact of installing an RTLS system at the hospital ward: assuming the RTLS systems functions as specified, there will never be any equipment that is not accounted for (however, RTLS does not guarantee availability).
The question of how many pieces of equipment to keep in stock can then be answered by setting two exogenous parameters: (1) the maximum permissible wait time d for the equipment, and (2) the maximum allowable probability, α, that a patient will not receive the equipment within the time d. Then we define the patent service level as $SL(k,d) = 1-\Pr\{W(k)>d\}=\Pr\{W(k) \leq d\}$, where W(k) is a random variable denoting the patient's waiting time in queue, if there is a single queue and k parallel servers.
The optimal amount of equipment to stock is the lowest k>0 such that the probability that a given arriving patient will wait for the equipment for more than d time units, is less than α:

$$\begin{aligned} &\min k \\ &\text{s.t. } \Pr\{W(k) > d\} < \alpha;\ k > 0 \end{aligned} \qquad (4)$$

Expressed in terms of patient service level, the optimization problem is

$$\min_{k>0} \{k \mid SL(k,d) > 1-\alpha\} \tag{5}$$

Now we can answer the two previous questions: how should the hospital optimally decide on the number of instances of each equipment type to own, and at what point should the hospital initiate a search? Using this inventory management system, a hospital would define both a desired patient service level (DSL), and a minimum acceptable patient service level (MSL). The amount of equipment needed is then computed as

$$N = \min_{k>0} \{k \mid SL(k,d) > DSL\} \tag{6}$$

Similarly, the optimal threshold for initiating a search is when K instances of equipment are accounted for (and therefore N-K are misplaced), where

$$K = \min_{k>0} \{k \mid SL(k,d) > MSL\} \tag{7}$$

The service level SL(k,d) depends on the underlying queuing model that is used to approximate reality. For simplicity of exposition, we will use an M/M/k queuing model here, which assumes that both inter-arrival times (of patients needing a particular piece of equipment) and service times (how long patients use the equipment) are exponentially distributed. Other queuing models such as M/G/k and G/M/k can be used in the same manner in our model; however, the service level equation becomes more complex.

Proposition 2. For an M/M/k queue, the patient service level SL(k,d) is given by

$$SL(k,d) = 1 - \gamma e^{-k\mu(1-\lambda/k/\mu)d} \tag{8}$$

where μ is the service rate for each equipment type and λ is the patient arrival rate at each equipment type and γ and P_{k-1} are defined as follows:

$$\gamma = \frac{\lambda(k\mu)^{-1}P_{k-1}}{1-\lambda(k\mu)^{-1}};\ P_{k-1} = \frac{\frac{(\lambda/\mu)^{k-1}}{(k-1)!}}{\sum_{i=0}^{k-1}\frac{(\lambda/\mu)^{i}}{i!}+\frac{(\lambda/\mu)^{k}}{k!}\frac{k\mu}{k\mu-\lambda}} \tag{9}$$

Proof. We know (see, e.g., [13]) that for exponential service times and inter-arrival times:

$$W^*(k) = \begin{cases} 0, & \text{w.p. } 1-\gamma(k,\mu,\beta) \\ \mathrm{Exp}\big(k\mu(1-\beta)\big), & \text{w.p. } \gamma(k,\mu,\beta) \end{cases} \tag{10}$$

where $\beta = \lambda\,(k\mu)^{-1}$. Therefore, $\Pr\{W^* \le a\} = \Pr\{W^* \le 0\} + \Pr\{0 < W^* \le a\} = \Pr\{W^* \le 0\} + \Pr\{W^* > 0\}\Pr\{W^* \le a \mid W^* > 0\} = 1 - \gamma + \gamma(1-e^{-k\mu(1-\beta)a}) = 1 - \gamma e^{-k\mu(1-\beta)a}$, which proves the claim.

Thus, given the desired and minimum service level requirements, the optimal amount of equipment to stock as well as the optimal time point of searches can be calculated. We next present a numerical study with an application to a fictitious but representative hospital ward.

NUMERICAL STUDY

For our numerical example we consider a medium-sized emergency care facility and take infusion pumps as the type of equipment of interest. Infusion pumps need to be ready for usage by patients, and typically exactly one infusion pump is used by a patient, as opposed to multiple pumps. Thus, usage of an infusion pump can be modeled using our approach. For this numerical study, we assume that, on average, λ=15 patients arrive per hour who require an infusion pump, and that each patient uses the pump, on average, for μ=1 hour.

Figure 1 demonstrates how the service level SL(k,d) varies with the number of equipment with known location (k) and the permissible wait time (d). The graph suggests that patient service level is concave-increasing in both k and d. The optimal number of infusion pumps to stock for this emergency care facility can be determined from Equation (6) once the desired patient service level components (1- α and d) are fixed.

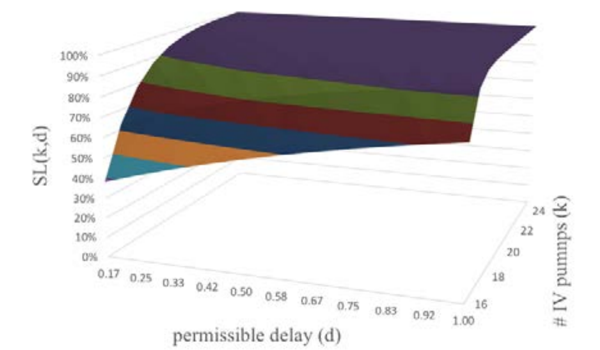

Figure 1: Patient service level vs. k and d

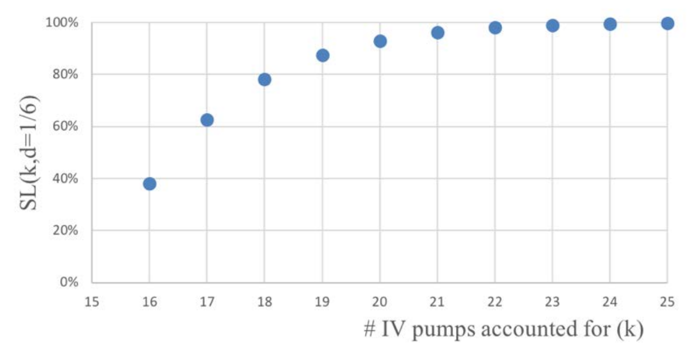

Figure 2: Patient service level vs. k

Figure 2 shows patient service level as a function of the number of equipment that is accounted for (Equation 8). In this example, the permissible wait time is chosen to be d=1/6, or 10 minutes. From the graph, or from Equation (8), we can see that if the facility needs to offer access to infusion pumps within less than 10 minutes for at least 95% of their patients (DSL=95%), then the optimal number of infusion pumps to stock (or procure) and have accounted for, is N=21.

Table 1 summarizes several key metrics in the tradeoff between patient service level and cost of directed search. For this example, we assume that the probability of misplacement is ω=0.2 per hour, the probability of accidental finds is σ=0.05 per hour, and there is a ψ=0.5 probability that one hour of directed search will locate all missing infusion pumps. Desired service level is 95%. Depending on the choice of minimum service level, the table shows the average service level SL= $\sum^{N}_{i=K} \pi_i$ SL(i,d); the average fraction of infusion pumps available AVL= 1/N $\sum^{N}_{i=K} i\pi_i$; the time between successive searches T; the expected number of times a directed search must be performed per year; and the expected cost of directed searches per year, DSC. For computing DSC, we assume a cost of search per hour of c=\$50, which is estimated from the hourly wage cost of a registered nurse.

MSL	K	SL [%]	AVL [%]	T [hours]	#Searches [year^{-1}]	DSC [\$ year^{-1}]
85%	19	94	97	11.3	779	77,900
70%	18	91	95	17.8	492	49,200
50%	17	87	92	19.5	450	45,000

Table 1: Key metrics for different minimum service level requirements

Table 1 illustrates the basic tradeoff: as the minimum service level decreases, the number of directed searches required over a given time horizon decreases. At the same time, the average service level also decreases.

We observe that as equipment gets misplaced, patient service level drops rapidly: in this example, once 4 (out of 21) pumps are misplaced, service level only satisfies the 50% minimum service level requirement. However, *average* service levels still appear reasonable even for K=17, because the system spends more time in the states where more equipment is accounted for (the steady state probabilities are not uniform across states). Nevertheless, the results show that when a minimum service level of 50% is required, the average patient service level is almost 10 percentage points below the desired service level. If the hospital wants to achieve an average patient service level close to the desired service level of 95%, they need to opt for a minimum service level of 85% or higher, along with its attendant higher directed search cost.

Table 1 makes abundantly clear that the frequency of directed searches is high, even for a very low minimum service level requirement. Even when planning with a comparatively low requirement of MSL=50%, the ward will on average perform a directed search more than once per day. This directly drives the directed search cost, which for MSL=85% is essentially equivalent to the budget for a full-time registered nurse. Even for MSL=50%, the cost of directed search remains economically significant at $45K per year.

CONCLUSION

In this paper, we develop an analytical framework for addressing the equipment misplacement problem specifically for healthcare facilities. Our framework combines two interlinked components: first, a queuing-based approach for determining the optimal equipment stocking levels as a function of required patient service levels; and second, a Markov-chain based approach for identifying the best directed search policy for recovering any misplaced equipment. As part of this approach, the hospital specifies both desired and minimum service levels for their patients.

When an RTLS is implemented to track the location of equipment, we can assume that, barring a malfunction of the RTLS, all of the equipment is continuously accounted for (which, as pointed out before, does not imply that the equipment is always available - availability is governed by demand and the queuing system in place). Thus, with RTLS, the desired service level is sustained once the optimal equipment stocking level, N, is set (Equation 6). Without RTLS tracking, misplacements will occur and patient service level will decrease over time. Therefore, if an RTLS is not used, a decision needs to be made at what point to initiate the directed search for misplaced equipment.

We conduct a numerical study to illustrate how to determine the optimal stocking levels and the optimal directed search policy in a practical hospital ward setting. The results of our numerical study are instructive for assessing the benefits of an RTLS implementation. We find that the costs incurred for directed searches alone appear to make a promising case for RTLS, especially when high minimum service levels are required: in our hypothetical example, in order to keep a minimum service level of 85%, the annual directed search cost amounts to approximately the fully-burdened salary of a registered nurse. It is important to note that our numerical example covers only a single type of equipment (infusion pumps); however, there are dozens of different types of equipment in any hospital ward that routinely get misplaced. Thus, we anticipate that the potential savings from RTLS in search cost avoidance alone, while likely not fully additive, will increase quickly the more equipment types that are tagged. Furthermore, the dominating cost factor for RTLS is likely to be the high fixed cost of mapping the locations, installing the readers, and customizing the software. The variable costs of RTLS for equipment tracking are likely comparatively low, because although the RTLS requires item-level tags, those tags only need to be affixed once to the equipment and then the equipment and tags circulate in a closed system. This setting is markedly different from the usual situation of item-level tagging in retail and distribution, where the variable cost of tags is taken to be the dominant cost factor; see, e.g., [5] and [9] for further discussion of the economics of item-level tagging.

In terms of future work, several opportunities exist. The queuing-based approach for determining optimal equipment stocking and patient service levels could be extended to include more complex queue types such as M/G/k or G/M/k, or G/G/k via approximation. In addition, further research could include a more detailed quantitative analysis of the payback period for RTLS implementations. Also of interest might be an attempt to identify what types of equipment likely show most promise for RTLS implementations: which equipment characteristics predict best the magnitude of benefit from RTLS? We also believe that, beyond the hospital setting, our methodology could be usefully applied to a wide range of systems where equipment needs to be available within a certain time period to provide service, e.g. at industrial plants.

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