THE IMPACT OF DYNAMIC VOLATILITY ON THE ESTIMATION OF THE DAY-OF-THE-WEEK EFFECT

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ABSTRACT

A well-known result in statistics is that least-squares estimators are sensitive to heteroscedasticity, and previous research has found that accounting for volatility changes can lead to dramatic changes in the magnitudes, and even in the signs, of the estimates of the mean levels of returns within financial time series. The current paper uses the estimation of the day-of-the-week effect to further explore the relationship between presumed volatility dynamics for financial time series and the estimation of the daily means under three volatility specifications – constant volatility, GARCH, and multifractal volatility – and finds, for example, that an apparent negative Monday effect may actually be a volatility rather than a mean effect.

Keywords: Day-of-the-week effect; multifractal volatility; GARCH; robust estimation.

EXTENDED ABSTRACT

GARCH and other dynamic volatility models have gained widespread acceptance in finance and even garnered a Nobel Prize, but their use has typically been focused on forecasting volatility and improving option pricing. On the contrary, lthough it is well-known result in statistical modeling that least-squares estimators are sensitive to heteroscedasticity (see, e.g., [2]), the implications of this sensitivity for financial returns, which are often subject to dramatic changes in volatility, have not been fully explored. Recently, though, in a study of 11 financial time series, [1] finds that accounting for changes in volatility through the use of the self-excited multifractal (SEMF) volatility model leads to dramatic changes in the magnitudes, and even occasionally in the signs, of the estimates of the mean returns for these series. Thus, changing volatility can clearly have a substantial impact on the empirical results obtained for other moments of the distributions of financial returns.

Unfortunately, the true volatility dynamics of the of financial time series are never known and instead must be estimated by various possible volatility models. The current paper extends the previous research in two directions by (1) incorporating a day-of-the-week effect in the estimates for the mean, and then (2) estimating these mean returns under three different sets of volatility specifications – constant volatility, GARCH-type volatility, and multifractal volatility. Historically, a typical finding with regard to the day-of-the-week effect has been for the average Monday return to be negative and lower than the average returns for the other four days, which tend to be positive. This paper will examine the impact of incorporating three different sets of volatility specifications – constant volatility, GARCH-type volatility, and multifractal volatility – on the estimates of the direction and magnitude of the components of the day-of-the-week effect in order to assess the importance of taking volatility dynamics into account when analyzing the level of financial returns.

In the initial analysis for this study, a day-of-week-mean model:

$$R_t = \beta_{Mon}1_{Mon} + \beta_{Tue}1_{Tue} + \beta_{Wed}1_{Wed} + \beta_{Thu}1_{Thu} + \beta_{Fri}1_{Fri} + \beta_{Hol}1_{Hol} + \varepsilon_t, \qquad (1)$$

is fit to daily S&P 500 returns and daily Nikkei returns for the period from January 2005 through December 2014 and is modeled jointly with one of three possible sets of volatility dynamics – constant volatility ($\sigma_t^2 = \sigma^2 \; \forall t$), GARCH volatility ($\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$), and SEMF volatility ($\sigma_t^2 = \sigma_0^2 e^{-\frac{2}{\sigma_0}\sum_{\tau=0}^{t-1}[h_0 e^{-\varphi(t-\tau-1)}]\varepsilon_\tau}$. The preliminary results obtained from fitting these sets of models are shown in the table below:

	S&P 500 Returns			Nikkei Returns		
	Constant	**SEMF**	**GARCH**	**Constant**	**SEMF**	**GARCH**
Monday	-0.0013	**0.0640**	0.0472	-0.0117	0.0549	0.0847
Tuesday	0.0993	**0.1178**	**0.1023**	-0.0169	-0.0696	0.0182
Wednesday	0.0377	**0.1166**	**0.1307**	0.0503	0.0640	0.0677
Thursday	0.0222	**0.1195**	**0.0818**	0.0896	**0.1288**	**0.1732**
Friday	0.0201	**0.1028**	**0.0931**	-0.0187	0.0612	0.0302
Post-Holiday	0.0266	0.0266	0.0325	**0.1377**	**0.2464**	**0.1474**

Clearly, changing the volatility assumption has a substantial impact on the estimated values for the various weekdays' means. For the S&P 500, the estimated Monday effect switches from negative to positive and becomes significant under the SEMF model. Tuesday's through Friday's all increase in magnitude and become significant. For the Nikkei, under constant volatility, Monday's, Tuesday's, and Friday's returns all appear negative (though insignificant), but only Tuesday's return remains negative under the SEMF specification and all appear positive under the GARCH specification. Interestingly, the values from both dynamic volatility specifications tend to be closer to each other, and to the days' median returns, as well, than they are to the values estimated assuming constant volatility. Thus, jointly modeling the mean with the variance has a clear impact on the estimated day-of-the-week effect, suggesting, e.g., that an estimated negative Monday effect is more a reflection of a volatility effect that was greatest on Mondays than on a tendency for Monday returns to be negative in general. Another equivalent and interesting finding that space precludes exploring more fully is that estimating the volatility and mean dynamics jointly can lead to substantively different estimated volatility coefficients than what is obtained from fitting these volatility dynamics to pre-whitened returns.

REFERENCES

[1] Ammermann, P. (2016) Are Stock Return Dynamics Truly Explosive or Merely Conditionally Leptokurtic? A Case Study on the Impact of Distributional Assumptions in Econometric Modeling. *Journal of Data Analysis and Information Processing*, 2016.

[2] Spanos, A. (1999) Probability Theory and Statistical Inference: Econometric modeling with observational data, Cambridge University Press, Cambridge, UK.