

# OPTIMIZING THE PERFORMANCE OF A SUPPLY CHAIN USING GREY-BASED TAGUCHI METHOD

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## ABSTRACT

This paper demonstrates the Grey-based Taguchi (G-Taguchi) method to solve a multi-objective optimization problem in the supply chain with the order-up-to-level  $S$  policy. A simulation model is run based on  $L_{18}$  Taguchi's orthogonal array design. The three-performance measures—order bullwhip effect (BWE), inventory BWE, and average fill rate—and the four controllable factors such as the information sharing rate, lead-time inflation constant, smoothing constant in forecasting, and inventory adjustment time are analyzed. G-Taguchi method generates a well-balanced solution with consideration of all three measures.

**Keywords:** bullwhip effect, fill rate, information sharing, grey relational analysis.

## INTRODUCTION

Over the last two decades, researchers have spent a considerable amount of time to identify the role and impact of information sharing in a supply chain. The order bullwhip effect (BWE)—the phenomena that a small change in a customer order creates a progressive increase in order variation as the order information passes upstream in the supply chain—has been particularly focused and studied using diverse system dynamics simulation models to identify its relationship with the information sharing. Many studies generally conclude that information sharing plays an important role in improving the performances of supply chains by reducing the order-related variations within the chains [3][4]. In addition, other measures such as the inventory variation (e.g., inventory BWE) and average fill rate (AFR) have been also frequently investigated. However, very few studies consider the balance among the performance measures. We note that one of the major challenges for supply chains is to provide a high fill rate at the low operating cost. For this, organizations need to minimize BWE's, and simultaneously need to maintain an appropriate fill rate to increase their revenue. To manage the balance among these performance measures, we need to identify the controlling factors for the measures and decide the optimal levels. In this study, we develop a three-echelon supply chain simulation model where each echelon has the same order-up-to-level  $S$  (OUT- $S$ ) policy. Information sharing between the echelons as well as the operating parameters of the OUT- $S$  policy is considered as a *controllable* operational factor. The three system-level performance measures—order BWE, inventory BWE, and AFR—are evaluated in terms of the four factors. To identify the optimal operational factor levels for the three measures, Taguchi's robust orthogonal design is used. We call optimal factor levels for a performance measure as a solution. The traditional Taguchi method is applied to identify optimal factor levels for each of the performance measures, individually without consideration of other performance measures. Thus, each solution is dedicated for its own performance measure. However, the optimal factor levels for one performance measure may not be optimal for other performance measures since each performance measure may have a different quality characteristic. Thus, the Grey-based Taguchi (G-Taguchi) method is suggested to identify *balanced*

optimal factor levels with consideration of all three measures, simultaneously.

## LITERATURE REVIEW

A considerable amount of research demonstrates that information sharing generally reduces the order BWE in a supply chain. Dejonckheere et al. [6] compare the order-up-to policy with the smoothing replenishment order (SRO) policy in a four-echelon serial supply chain model. They show that the order BWE increases at both OUT- $S$  and the OUT smoothing replenishment order policies when the echelons do not share retailer's forecast information. With the forecast information shared, the reduction of the order BWE is more significant in case of the SRO policy. Geary et al. [7] emphasize that order BWE cannot be eliminated from supply chains, but it can be minimized. Barlas and Gunduz [2] evaluate the inventory BWE under three different order policies: an OUT- $S$ , an anchor-and-adjust, and an order-point  $s$ , order-up-to- $S$  level, referred to as  $(s, S)$  policy, and demonstrate that sharing the retailer's forecast significantly reduces the inventory BWE across all order policies. Costantino et al. [5] analyze the order BWE and inventory stability using a four-echelon model. Jeong and Hong [8] identify that a high level of information sharing and a high level of information balance between echelons reduce the order BWE while the first is more significant than the latter. They also show that a significant information unbalance between echelons may cause the reverse BWE using the four-tier supply chain model.

In our case, we are applying the G-Taguchi method to the supply chain model to optimize its performance in terms of the multiple measures under different demand conditions. It incorporates the Grey Relational Analysis (GRA) into the Taguchi method to transform a multi-objective problem into the single objective problem and optimize it using the Taguchi method. Kuo and Yang [10] propose the G-Taguchi method to optimize a multi-objective simulation problem and illustrate it. Many researchers apply the GRA to the supplier section problems by integrating GRA with other multi-criteria decision making (MCDM) methods [9][13][12]. Banasik et al. [1] identify that the use of MCDM approaches for designing green supply chains is not a new but emerging research field.

## SUPPLY CHAIN MODEL

The three-echelon supply chain simulation model consists of a retailer, a distributor, and a producer. At each period  $t$ , each echelon,  $i$  ( $i = 1, 2, 3$  for the retailer, distributor, and producer, respectively), receives an order from its immediate downstream echelon,  $i-1$ , and attempts to satisfy an incoming order as soon as possible using its own available on-hand inventory, and then issues a replenishment order to its immediate upstream echelon,  $i+1$ . If an echelon does not have enough inventory, a partial fulfillment of the order is allowed, and the remaining portion of the order is backlogged without any loss. Each echelon uses the simple exponential smoothing (SES) forecasting method and the same OUT- $S$  policy. The logic of the traditional beer simulation game is adopted (See the detailed logic at [2]).

The demand estimate,  $\hat{d}_t^i$ , at the  $i^{\text{th}}$  echelon at time  $t$  is represented as in Equation (1). If the forecast at retailer,  $\hat{D}_t^1$ , is shared through collaboration at other echelons (that is, the information sharing rate ( $ISR$ ) at that echelon is set to 1), then, the demand estimate at the echelon,  $\hat{d}_t^i$ , becomes  $\hat{D}_t^1$ . Otherwise (that is,  $ISR = 0$ ),  $\hat{d}_t^i$  is set to  $\hat{D}_t^i$ , the demand forecast by the SES forecasting method since the echelon needs to forecast its demand independently.

$$\hat{d}_t^i = (1 - ISR) * \hat{D}_t^i + ISR * \hat{D}_t^1 \quad (1)$$

The order quantity,  $O_t^i$ , is determined by the OUT- $S$  policy as in Equation (2). Note that the order quantity at  $i^{\text{th}}$  echelon becomes a demand for the immediate upstream echelon while the shipment quantity from the upstream echelon is delivered to the downstream echelon. In this way, the information (order) flows

forward while the material (shipment) follows backward.

$$O_{i,t} = \max((SL_t^i - IP_t^i)/IAT, 0) \quad (2)$$

, where  $SL_t^i = (LT^i + K) \hat{d}_t^i$  is the order-up-to level, represented by the lead-time between echelons,  $LT^i$ , the lead-time inflation factor,  $K$ , and the demand estimate,  $\hat{d}_t^i$ .  $IP_t^i$  refers to Inventory Position, the sum of the on-hand inventory, inventory in the pipeline from the upstream echelon less the outstanding orders at the echelon.

We use three system-level performance measures—order BWE ( $BWE_o^s$ ) in Equation (3), inventory BWE ( $BWE_l^s$ ) in Equation (4), and AFR ( $AFR^s$ ) in Equation (5), respectively.

$$BWE_o^s = \frac{\sigma_{s,o}^2/\mu_{s,o}}{\sigma_d^2/\mu_d} \quad (3)$$

$$BWE_l^s = \frac{\sigma_{s,l}^2/\mu_{s,l}}{\sigma_d^2/\mu_d} \quad (4)$$

, where  $\mu_{s,o}$  and  $\sigma_{s,o}^2$  are the mean and the variance of the system-level order quantity ( $O_t^s$ ), respectively, while  $\mu_{s,l}$  and  $\sigma_{s,l}^2$  are the same metrics for the system-level inventory level ( $I_t^s$ ), respectively. Note that  $\mu_d$  and  $\sigma_d^2$  are the mean and the variance of the customer order,  $d_t^1$  in Equation (1).

$$AFR^s = \sum_i \frac{1}{T} \sum_t \frac{(S_t^1 - B_t^1)}{d_t^1} Id_{(S_t^1 - B_t^1)} \quad (5)$$

, where  $S_t^1$  and  $B_t^1$  are the shipment and backorder, respectively at the retailer.  $Id_{(x)}$  is an indicator variable defined to 1 only if  $x \geq 0$ , and  $T$  is the total simulation time; 0 otherwise.

## TAGUCHI DESIGN AND ANALYSIS

The three system-level performance measures are evaluated in terms of the four operational factors listed in Table 1. Based on this information, we use  $L_{18}$  Taguchi's orthogonal array design, which generates the eighteen different alternatives listed in Table 2. With the  $L_{18}$  alternatives, the system dynamic simulations are conducted for the two different cases of the demand, normal (20, 2) and normal (20, 4) where 'normal' stands for the normal distribution, and the first and second numbers represent the mean and the standard deviation of the normal distribution. Each alternative has 25 repetitions for each case. Demand is considered as an uncontrollable factor (a noisy factor). The lead-time between echelons is set to 3 time units and the model is run for 365 time units.

Control Factor	Control Factor	No of Level	Control Factor values
$ISR$	Information sharing rate	2	0, 1
$K$	Lead-time adjustment constant	3	2, 3, 4
$\alpha$	smoothing constant in the SES forecasting	3	0.2, 0.4, 0.6
$IAT$	Inventory adjustment time	3	0.6, 0.8, 1.0

Table 1. Level and Notation of the Control Factors in  $L_{18}$  Design

Thus, there are total 50 repetitions for each alternative. The Signal-to-Noise (SNR) is a measure of robustness used to identify control factors that reduce variability in a given system by minimizing the effects of uncontrollable factors (two demand cases in this study). We calculate the SNR for each of the three performance measures. Taguchi defines a different SNR formula for a performance measure with a different quality characteristic—smaller-the-better (e.g., BWE) in Equation (6) or larger-the-better (e.g., AFR) in Equation (7) below.

$$SNR(LB) = -10\log_{10}\left(\frac{1}{n}\sum_{l=1}^n 1/r_l^2\right) \quad (6)$$

$$SNR(SB) = -10\log_{10}\left(\frac{1}{n}\sum_{l=1}^n r_l^2\right) \quad (7)$$

, where  $n$  is the number of responses observed, 50 in this study, and  $r_l$  is the  $l^{th}$  observed response value. Higher values of the SNR identify optimal factor levels that minimize the effects of the noise factors. The SNR of each performance measure is calculated in Table 2. The main effect analysis is applied to identify the optimal factor levels. The optimal factor levels or solution for the order BWE, denoted by Opt (order BWE), are  $ISR(2)-K(1)-\alpha(1)-IAT(3)$ , where the number within a parenthesis is the level of the factor. Those for inventory BWE, represented by Opt (inv BWE), and for AFR, represented by Opt (AFR), are  $ISR(2)-K(2)-\alpha(1)-IAT(2)$  and  $ISR(1)-K(3)-\alpha(2)-IAT(1)$ , respectively. The graphs of the optimal factor levels are as displayed in Figures 1(a), 1(b), and 1(c) for each of the performance measures, respectively. We can observe that there are no consistent optimal factor levels performing well across all measures.

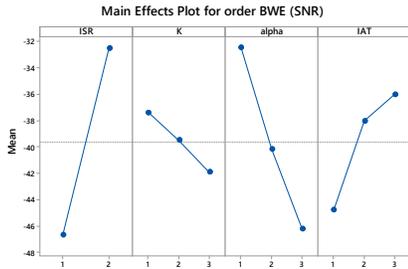


Figure 1(a). Optimal Factor Level for order BWE

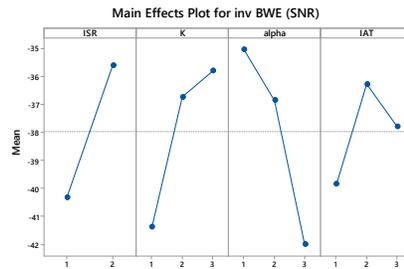


Figure 1(b). Optimal Factor Level for inventory BWE

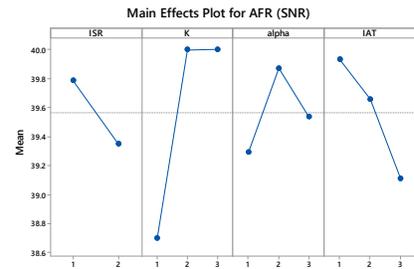


Figure 1(c). Optimal Factor Level for AFR

## GREY BASED TAGUCHI METHOD

Each of the three solutions obtained previously is not the balanced optimal level. Therefore, we apply the G-Taguchi method to derive a balanced solution. The GRA in the G-Taguchi is a process of transforming all performance measures into a single comparable measure, called Grey Relational Grade (GRG).

The GRA is briefly described below.

*Step 1: Normalizing the performance measures for fair comparison*

Since each performance measure has a different range of the value, all values are transformed into normalized values between 0.0 and 1.0 at each alternative. Let  $y_j(k)$  be an observed value of the  $k^{th}$  performance measure ( $k = 1, \dots, 3$ ) at the  $j^{th}$  alternative ( $j = 1, \dots, 18$ ). The normalized value,  $x_j(k)$ , is calculated by Equation (8)

$$x_j(k) = \frac{y_j(k) - \min\{y_j(k), \forall j\}}{\max\{y_j(k), \forall j\} - \min\{y_j(k), \forall j\}}; \forall j \quad (8)$$

*Step 2: Reference sequence*

A reference sequence is a normalized set of target performance values between 0 and 1, where 1 is the highest target value for all performance measures. Therefore, our reference sequence,  $x_0(k)$ , is (1, 1, 1) in the order of order BWE, inventory BWE, and AFR.

*Step 3: Grey relational coefficient*

The Grey Relational Coefficient (GRC) determines how close a normalized value,  $x_j(k)$ , is to the corresponding reference value,  $x_0(k)$ . It is defined in a way that the closer  $x_j(k)$  and  $x_0(k)$  are, the larger it is. The GRC between  $x_j(k)$  and  $x_0(k)$  is defined by

$$\gamma(x_0(k), x_j(k)) = \frac{\Delta_{min} + \zeta \Delta_{max}}{\Delta_{j_0}(k) + \zeta \Delta_{max}}, \forall j \forall k \quad (9)$$

, where  $\Delta_{j_0}(k)$  is the absolute distance between  $x_j(k)$  and  $x_0(k)$ , which is defined as  $|x_j(k) - x_0(k)|$ . The  $\Delta_{min}$  and  $\Delta_{max}$  are minimum and maximum of  $\Delta_{j_0}(k)$ , respectively for all  $j$ 's and  $k$ 's. After grey relational generation using Equation (8),  $\Delta_{min}$  and  $\Delta_{max}$  will be equal to 0 and 1, respectively.  $\zeta$  is a distinguishing coefficient,  $\zeta \in [0, 1]$ , which expands or compresses the range of the GRC. In this study,  $\zeta$  is set to 0.5.

#### Step 4: Grey Relational Grade

After the GRC, we determine the weights of the performance measures to calculate the GRG. The multiplication of the GRC with the weights generates a single GRG. We use the correlation matrix method [11], generating (0.368, 0.375, 0.275) for order BWE, inventory BWE, and AFR, respectively. The final GRG values are listed in Table 2. Readers can refer to [10] for detailed information on GRA.

Note that Opt (AFR) is not in the L<sub>18</sub> design. The main effect analysis identifies the optimal solution, denoted by Opt(GRG), *ISR* (2)-*K*(3)- $\alpha$ (1)-*IAT*(2), displayed in Figure 2.

Alter	Factor level				Average			GRG
	<i>ISR</i>	<i>K</i>	$\alpha$	<i>IAT</i>	O-BWE	INV-BWE	AFR	
1	1	1	1	1	-41.172	-38.440	39.821	0.689
2	1	1	2	2	-43.668	-39.848	39.477	0.627
3	1	1	3	3	-45.861	-41.003	38.751	0.552
4	1	2	1	1	-43.681	-35.751	40.000	0.727
5	1	2	2	2	-47.167	-39.852	39.999	0.684
6	1	2	3	3	-49.213	-41.236	39.997	0.668
7	1	3	1	2	-37.407	-30.290	40.000	0.818
8	1	3	2	3	-44.789	-36.282	40.000	0.719
9	1	3	3	1	-67.356	-60.355	40.000	0.558
10	2	1	1	3	-22.282	-45.388	35.934	0.598
11	2	1	2	1	-36.415	-39.879	39.748	0.691
12	2	1	3	2	-35.190	-43.697	38.457	0.568
13	2	2	1	2	-26.298	-33.127	39.998	0.862
14	2	2	2	3	-29.849	-35.547	39.984	0.803
15	2	2	3	1	-40.953	-34.915	40.000	0.746
16	2	3	1	3	-24.272	-27.290	40.000	0.974
17	2	3	2	1	-39.093	-29.730	40.000	0.818
18	2	3	3	2	-38.608	-30.861	40.000	0.804

Table 2. Grey Relational Computations

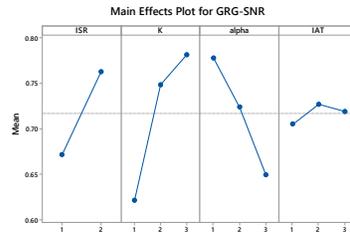


Figure 2. Optimal Factor Level for GRG

These results from the G-Taguchi method are analyzed by the analysis of variance (ANOVA) to understand the significance and impact of each operational factor on the supply chain performance, represented by the GRG, in Table 3. The coefficient of determination,  $R^2$ , of the model is 78.01%. Three factors—*ISR*, *K*, and  $\alpha$ —are statistically significant. The result is well aligned with Figure 2. Among these, *K* has the highest impact on the explanation of the total variation (38.43%);  $\alpha$  has the second highest

impact (22.22%), while the *ISR* has 16.70% of the contribution percentage. Based on Figure 2 and Table 3, we can say that the supply chain performance improves with information shared (*ISR* = 1), more safety stock through the lead-time inflation factor (*K* = 3), and smoother forecast in this study.

Source	DF	Adj SS	Adj MS	F-Value	p-value	Percentage of Contribution
<i>ISR</i>	1	0.037585	0.037585	7.59	0.02*	16.70%
<i>K</i>	2	0.086509	0.043255	8.74	0.006**	38.43%
$\alpha$	2	0.050009	0.025005	5.05	0.03*	22.22%
<i>IAT</i>	2	0.001495	0.000748	0.15	0.862	0.66%
Error	10	0.049495	0.00495			21.99%
Total	17	0.225094				100.00%

\* *p* value < 0.05, \*\* *p* value < 0.01

Table 3. ANOVA for GRG

### CONFIRMATION TEST

Now, we compare the balanced solution with three other dedicated solutions. For this, we use the same simulation model and conditions, and collect all statistics again. Now, we display the average of the performance measures at each optimal solution in Table 4 with the corresponding GRG values. We see that the first three solutions optimize their dedicated performance measure at the cost of other performance measures. For example, Opt (order BWE) optimizes the supply chain in terms of the order BWE (12.714; the smallest BWE), but it really has a high inventory BWE and small AFR value. However, the balanced solution, Opt (G-Taguchi), generates a reasonable value for all performance measures. It even has the best values for inventory BWE and AFR with the highest GRG among all solutions (see Deviation to see the deviation percentage from the best value), indicating that the G-Taguchi method performs well.

Solution	Optimal Level				Performance Measures			GRG
	<i>ISR</i>	<i>K</i>	$\alpha$	<i>IAT</i>	order BWE	inv BWE	AFR	
Opt (order BWE)	2	1	1	3	12.714*	166.182	84.880	0.734
Opt (Inv BWE)	2	3	1	2	20.325	41.616	99.977	0.980
Opt (AFR)	1	3	2	1	717.210	307.751	100.000*	0.500
Opt (G-Taguchi)	2	3	1	2	23.188	19.556*	100.000*	0.994
Deviation (%)					82.38	0	0	

\* best value

Table 4. Confirmation Results.

### CONCLUSIONS

We observe that the traditional Taguchi method optimizes one performance measure at a time at the cost of other performance measures, while the G-Taguchi method identifies the balanced optimal factor levels with consideration of all performance measures. The balanced solution even generates the best value for the inventory BWE and AFR. Overall, a higher information sharing rate, a higher lead-time inflation factor, and a smoother forecast contribute to a higher GRG value, showing a linear or piece-wise linear pattern. However, in case of the inventory adjustment time, a non-linear impact is observed.

### REFERENCES

References available upon request from *Ki-Young Jeong, jeongk@uhcl.edu*