VENDOR MANAGED INVENTORY (VMI) UNDER A LEADER-FOLLOWER FRAMEWORK WITH HETEROGENEOUS RETAILERS

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ABSTRACT

This paper studies the effectiveness of incentive-aligned vendor-managed inventory (VMI) system in a supply chain where the vendor supplies heterogeneous retailers under a sequential game theoretic framework. The vendor takes the role of a Stackelberg leader, while the multiple retailers act as followers. In such a setting, the paper investigates how fixed shipping cost allocation, wholesaler price, and delivery frequency can be determined between the vendor and retailers. Through game theoretic analysis of the model and extensive numerical experiments, we obtain that the wholesale price determined in the VMI contract is contingent on the shipping cost allocation scenario as well as on the heterogeneity of retailers in terms of market shares.

Keywords: Vendor-managed inventory (VMI), Supply chain management, Stackelberg game, Shipping cost allocation, Simulation

INTRODUCTION

In this age of globalization and outsourcing, most firms control only a small part of the overall supply chain of any products. Therefore, to ensure that the value of the supply chain is maximized, all upstream and downstream firms need to actively coordinate operations and align their supply chain strategies. Vendor-managed inventory (VMI) system is one such initiative that helps to facilitate this coordination.

In a VMI contract, the vendor makes replenishment and inventory decisions for its downstream buyers (e.g., retailers) with real time access to end level demand information from the retailers. With access to this information, the vendor at the upstream of the supply chain is deemed to be in a better position to make optimally integrated decisions for the entire supply chain. In contrast, in the standard retailer-managed inventory (RMI) system, order quantities, order frequency, and inventory decisions are all controlled by the retailers. In the RMI system, replenishment decisions are made by each retailer independently of each other in a decentralized manner without coordination efforts between retailers and the vendor. Here the vendor simply responds to individual retailers' orders by filling and shipping their requested order quantity without having access to the final market demand information

In general, benefits of VMI include reduced inventory holding across the system, improved product availability (e.g., cycle service levels and fill rates), and enhanced relationships with business partners via transparent communications and information symmetry in the supply chain (Achabal et al. [1], Yao and Dresner [5]).

There is an extensive literature on VMI, a detailed survey of which is available in Marquès, Thierry, Lamothe and Gourc [4]. However, application of game theoretic models to study VMI is a more recent phenomenon, rich in contents and approaches in developing ways to coordinate in the supply chain. It has been common in this body of literature to set up the supply-chain model assuming the vendor is the Stackelberg leader, and one or more retailers being the followers, starting with Yu et al [7], and Yu et al [6]. These models, and many other research in this area, focus on how the optimal retail price and market demand for the retailers are affected due to the efficiency caused by the VMI system. Almehdawe and Mantin [2], other than considering the standard situation where the vendor is the Stackelberg leader, also extended their analysis by considering a situation where one of the retailers as the leader. By comparing the two cases, they found that in general higher supply chain efficiency is ensured when the retailer is the Stackelberg leader.

In most of the existing VMI literature based on a Stackelberg leader-follower model, the efficiency across the supply chain stems from stimulating the final demand by varying the retail price. However, in situations where the overall market demand is steady, or the product is less price-sensitive, the focus should be put on enhancing coordination across the supply chain by aligning incentives among business partners. Our objective is to study the efficiency of the supply chain under such a setup, where the possibility of improving the efficiency of the VMI system is identified based on sequential leader-follower Stackelberg game. The approach implemented in this paper to enable common shipments to be made from the vendor to the retailer pool generates substantial fixed shipping cost savings and has not been used with Stackelberg game in existing literature to the best of our knowledge.

OBJECTIVE AND RESEARCH QUESTIONS

The objective of this paper is to assess the benefits of a VMI partnership between *a vendor and multiple retailers*, which is both relevant and realistic. We consider a sequential game theoretic framework, with the vendor as the Stackelberg leader, and the multiple retailers being the followers. Furthermore, in the VMI system that we examine, a nested ordering policy is considered to utilize the benefits of cross-docking practice. Our focus is to determine how the optimal VMI contract depends on the fixed shipping cost allocation, wholesale price, delivery frequency and heterogeneity among retailers. In real life situations, the vendor and the retailers negotiate on shipping terms, and there is no clear industry standard with respect to the fixed shipping cost allocation between them. For the purpose of this study, we focus on the scenario in which fixed shipping costs are shared between the vendor and retailers using a game theoretical approach.

The aim of this paper is to address the following research questions:

- How do the design parameters (e.g., number of retailers) in the system affect supply chain performance when a vendor managed inventory (VMI) system is implemented? How do these parameters determine whether a VMI system is more profitable than a RMI system?
- What are the shipping cost allocation terms under which a VMI is beneficial for all stakeholders? What are potential limitations of VMI implementation in such a setting?
- How do the optimal wholesale price, shipping cost allocation and delivery frequency interact with each other to achieve a coordinated VMI contract between the vendor and buyers?

Using numerical methods, this paper considers a range of contingencies not captured in previous research on VMI (Yao, Dong, & Dresner [5]). Initial results indicate that the wholesale price determined in the VMI contract is contingent on the shipping cost allocation scenario as well as on the heterogeneity of retailers in terms of market shares.

THE BASIC MODEL

As stated earlier, this paper considers a supply chain with a single vendor who caters to multiple retailers, say n. Each retailer faces steady and deterministic demand, and the vendor, in turn replenishes the item from an external supplier.

The notations used in this paper are listed below. At retailer i, where $i = 1,2,\ldots,n$:
D_i: Demand faced by retailer i.
F_i^{VMI}: Fixed shipping charge, allotted to retailer i under VMI.
F_i^{RMI}: fixed shipping charge allocated to retailer i under RMI.
k: holding cost rate.
S_i: fixed ordering cost charged to retailer i.
Q_i: order quantity/order for retailer i under *RMI.*
Q_i^{VMI}: each shipment quantity from the vendor to retailer i under VMI.
t_i: Retailer i's order cycle, under RMI.
w: list wholesale price (not discounted).
w_D: discounted wholesale price.

At the vendor:
c_V: purchasing cost/unit incurred (paid).
F_v^{VMI}: Fixed shipping charge, allotted to vendor under VMI.
F_v^{RMI}: fixed shipping cost charged to vendor under RMI.
π_V^{VMI}: Vendor's total revenue under VMI
S: fixed ordering cost charged to vendor.
T: Vendor's own order cycle.
y: Number of shipments made to retailers during time T, the vendor's order cycle, under VMI.
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For the whole system, the following notations are defined:
$D = \sum_{i=1}^{n} D_i$, total market demand, obtained from the sum of the demand faced by each retailer.
F^{RMI}: total fixed shipping cost under RMI.
F^{VMI}: total fixed shipping charge under VMI, which needs to be shared between the vendor and retailers.

A non-cooperative two-stage Stackelberg leader-follower game is assumed. In *stage 1*, the vendor, denoted V decides the number of shipments (y) and the discounted wholesale price, with the objective of maximizing profit. The vendor keeps in mind the optimal response of the retailers in stage 2, contingent to the shipment frequency (y), wholesale price discount and fixed shipping cost allocation that it chooses in stage 1. In *stage 2*, given the number of shipments (y), the discounted wholesale retail price, and fixed shipping cost allocation, the retailers decide whether to agree to the VMI contract, or decline the VMI contract and choose to stay with RMI system.

The Stage 2 Game: Since we solve two-stage Stackelberg games by backward induction, we need to solve the stage 2 game first, where the retailers choose their optimal strategy. In terms of our set up, this implies that, when offered the option of converting to a VMI contract, each retailer *i* will agree to convert if it is not worse off under the VMI system compared to the existing RMI system. Recall that the demand faced by the retailer is assumed to be fixed, as mentioned in the introduction. This, in turn means that, to agree to the VMI contract, for any retailer *i*, its total cost under RMI must be higher than that under VMI, i.e. $TC_i^{RMI} \geq TC_i^{VMI}$.

Given the notations in our supply chain model, retailer *i*'s cost under VMI and LMI are as follows:

$$VMI\text{: } TC_i^{VMI} = \left(\frac{Q_i^{VMI}}{2}\right) kw_d + \left(\frac{D_i}{Q_i^{VMI}}\right) F_i^{VMI} = \frac{TD_i}{2y} kw_d + \frac{y}{T} F_i^{VMI} \quad (1)$$

$$RMI\text{: } TC_i^{RMI} = \left(\frac{Q_i}{2}\right) kw + \left(\frac{D_i}{Q_i}\right)(S_i + F_i) \quad (2)$$

Using the expressions of the total costs under RMI and VMI in equations (1) and (2), we find that condition under which a retailer will move to a VMI contract, namely $TC_i^{VMI} < TC_i^{RMI}$, can be stated simply as:

$$F_i^{VMI} \leq \frac{1}{2} \cdot k \cdot \frac{T}{y}(w \cdot Q_i - w_D \cdot Q_i^{VMI}) + \frac{T}{y}\frac{1}{t_i}(S_i + F_i) \quad (3)$$

The expression for F_i^{VMI} above is the threshold value or upper bound of the fixed shipping cost for which retailer *i* will agree with the VMI offered by the vendor and constitutes our first key result.

The Stage 1 Game: We now move on to the *stage 1 game*. Here, the vendor sets the optimal VMI contract to maximize its profit, while ensuring that each retailer accepts to move to the VMI contract, i.e. condition (3) is satisfied for any retailer *i*. To accommodate every retailer to agree to the VMI system, the vendor needs to ensure that the fixed shipping cost allocated to the retailers does not exceed condition (3) for any given retailer. For this to happen, the fixed shipping cost to be allocated to each retailer, denoted F_R^{VMI} , must satisfy the following condition:

$$F_R^{VMI} = min(F_i^{VMI}), i = 1,2, \dots, n. \quad (4)$$

If the vendor charges the same fixed shipping cost to each retailer under VMI, then the shipping cost must be no more than $\mathrm{F_R^{VMI}}$ as defined above in equation (4) to ensure that all retailers agree to the VMI contract.

Further, based on the critical level of fixed shipping cost allotted to each retailer under VMI, which we determined to be F_R^{VMI} in equation (4), we define the corresponding quantity in each shipment from the vendor to retailer *i* under VMI to be Q_R^{VMI}, and the demand faced by each retailer to be D_R, where $Q_R^{VMI} = \frac{T.D_R}{y}$.

Recall that F^{VMI} denotes the total fixed shipping costs that needs to be shared by the vendor and *n* retailers. Using condition (4), we can express the fixed shipping cost allocation condition as $F^{VMI} = F_V^{VMI} + n.F_R^{VMI}$. This allows us to write the objective function of the vendor as

$$\text{Maximize } \pi_V^{VMI}(y) = Dw_D - \left[\frac{1}{T}S + \frac{y}{T}F_V^{VMI} + \frac{DT}{2y}(y-1)k.C_V\right] \quad (5)$$

To check the characteristics of the vendor's profit function, we take the second difference equation of $\pi_V^{VMI}(y)$ with respect to *y*, using equation (5) and derive the following two conditions:

(i) For $D.C_V > n.w_D.D_R$, π_V^{VMI} is convex with respect to y
(ii) For $D.C_V < n.w_D.D_R$, π_V^{VMI} is concave with respect to y

In case of (i), when π_V^{VMI} is convex with respect to *y*, it makes sense for the vendor to be shipping to retailers all at once (i.e., *y* = 1) during the vendor's own order cycle *T*.

Thus, the vendor maximizes profits as stated in equation (5) by selecting the largest value of discounted wholesale price w_D that satisfies condition (i).

In case of condition (ii), when the vendor's profit function is concave with respect to *y*, the optimal shipping frequency *y** within the order cycle *T* is the largest value of y that satisfies $\Delta\pi_V^{VMI} \geq 0$, which leads to the following equation for the optimal discounted wholesale price, w_D:

$$w_D = \frac{D}{D_R} c_V + y(y+1) \frac{2F^{VMI}}{k.T^2} \cdot \frac{1}{n.D_R} \quad (6)$$

NUMERICAL RESULTS AND MANAGERIAL INSIGHTS

Through numerical examples, we intend to examine how supply chain design parameters such as the number of retailers and the retailer heterogeneity (in terms of their demand size) influence determination of the discounted wholesale price as well as reallocation of fixed shipping costs amongst supply chain partners using a leader-follower game framework. Demand and cost parameters used for the numerical experiments are summarized in Table 1.

Table 1. Demand and cost parameters for numerical experiments

At retailer *i*	**At the vendor**
Demand at retailer *i* = as per retailer heterogeneity Holding cost rate, *k* = 20% per year Whole price (*w*) = \$30/unit Discounted wholesale price (w_D) = To be decided Retailer's shipping cost RMI (F_i^{RMI}) = \$200 Retailer's ordering cost RMI (S_i) = \$100 Retailer's shipping cost VMI (F_i^{VMI}) = To be decided	Manufacturing cost (c_V) = \$15/unit Vendor's ordering cost (*S*) = \$500/order Vendor's shipping cost RMI (F_v^{RMI}) = \$200 Vendor's shipping cost VMI (F_v^{VMI}) = To be decided **System** Total system demand (*D*) = 5,000 units/year # of retailers (*n*) = 1, 2, 4, 6, 8, & 10 Retailer heterogeneity = no, low, med, high

Results obtained on the impact of the number of retailers and the retailer heterogeneity indicate that the wholesale price discount and the fixed shipping cost allocation can be combined to induce the retailers to agree to the VMI contract. In particular, the greater the heterogeneity (in demand sizes) among retailers, the greater the expectations of financial incentives offered to retailers (both in terms of the wholesale price discount and fixed shipping cost savings). On the other hand, the impact of the number of retailers on deciding the discounted wholesale price does not show significance effects as revealed in Table 2. However, it should be noted that there is a significant decrease in the fixed shipping cost allocated to retailers (or cost savings) when the optimal shipment frequency within the vendor's reorder cycle, *T*, changes from $y^* = 2$ to $y^* = 1$. This can be explained by the increased benefits it generates for the vendor (i.e., fewer shipments and lower on-hand inventory) who is now willing to take smaller portion of the fixed shipping cost savings.

Table 2. Impact of design parameters on wholesale price and fixed shipping cost allocation

	# of Retailers					
No heterogeneity	**1**	**2**	**4**	**6**	**8**	**10**
W_D	\$29.5	\$28.4	\$28.3	\$28.4	\$28.3	\$28.4
F_R(VMI)	\$198	\$155	\$139	\$101	\$98	\$98
Shipment freq	2	2	2	1	1	1

	# of Retailers					
Low heterogeneity	**1**	**2**	**4**	**6**	**8**	**10**
W_D	\$29.5	\$27.0	\$27.1	\$27.0	\$26.9	\$26.8
F_R(VMI)	\$195	\$143	\$125	\$83	\$82	\$83
Shipment freq	2	2	2	1	1	1

	# of Retailers					
Med heterogeneity	**1**	**2**	**4**	**6**	**8**	**10**
W_D	\$29.5	\$26.8	\$26.8	\$26.9	\$26.8	\$26.8
F_R(VMI)	\$192	\$140	\$122	\$79	\$78	\$78
Shipment freq	2	2	2	1	1	1

	# of Retailers					
High heterogeneity	**1**	**2**	**4**	**6**	**8**	**10**
W_D	\$29.5	\$26.6	\$26.5	\$26.5	\$26.4	\$26.4
F_R(VMI)	\$190	\$131	\$115	\$107	\$65	\$64
Shipment freq	2	2	2	2	1	1

Even under such a favorable setting (i.e., a single shipment to be made to retailers within *T*) created for the vendor, the wholesale price discount is shown to be insensitive and unaffected by the change in the shipment frequency as shown in Table 2.

CONCLUDING REMARKS AND FUTURE RESEARCH

This paper investigates how the leader-follower Stackelberg model can be applied in a two-stage supply chain in identifying the right financial incentives for retailers (buyers) and the vendor. The paper develops an analytical model, where common shipments are made to the retailer pool within the vendor's order cycle. Using numerical simulations, the paper concludes that the wholesale price determined in the VMI contract is contingent on the shipping cost allocation scenario as well as on the heterogeneity of retailers in terms of market shares. Although the deterministic and steady demand setting faced by retailers is considered a limitation, this research can be expanded into future studies to incorporate a greater level of dynamics (retailer competition) and supply chain uncertainties. Further, the scope of business scenarios can be broadened to gain insights on other supply chain settings in which either the vendor or the retailers take on most of the fixed shipping costs as per Incoterms.

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