ROBUST STRUCTURAL EQUATIONS FOR SAFETY STOCK OPTIMIZATION IN MULTI-ECHELON SUPPLY CHAIN NETWORKS

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ABSTRACT

Intense competition in e-commerce has forced firms to provide highly precise delivery time guarantees such as 'same-day' and 'two-day' shipping. In order to meet these stringent standards, it is imperative that companies avoid stock-outs across multi-echelon supply chain networks. Present approaches for optimizing network-wide safety stock levels are challenging tools to use for day-to-day managerial decision making, due to their demanding computational requirements. We propose a methodology that creates structural equations using regression techniques, based on known optimal results. The procedure is useful during situations such as initial decision-making on network-wide safety stock levels where quick, robust answers are sufficient.

Keywords: Multi-echelon supply chains, safety stock, supply chain management, e-commerce, structural equations modeling

INTRODUCTION

Ensuring timely delivery of products to their customers and avoiding stock-outs has always been a prime objective of companies. But with the ever-increasing competition for market share and the advent of e-commerce focused business models over the last two decades, this has come to the forefront of competitive priorities for organizations across many industries. Amazon, for example, has converted speedy delivery to one of its main strategic tools, spending $ 11.5 billion (10.8% of its revenue) on shipping in 2015 [22].

Taking a step back, at the organizational level, achieving target service times is an extremely complicated task. This is because real-world supply chains comprise of several stages, where each stage is associated with a process such as procurement, manufacturing or transportation of items [4]. Achieving target service times consistently involves overcoming various challenges and uncertainties of demand level, lead times and suppliers across all stages. The Beauty & Grooming global business unit of Proctor & Gamble (P&G), as an example, has 4,000 to 5,000 such stages and 6,000 to 10,000 inter-connections between them (referred to as 'arcs') to process a product line of approximately 500 finished goods [5]. While having numerous stages increases the risks of not being able to meet target service times, they also help mitigate this risk by serving as potential points of holding safety stock inventory. This enables each stage to meet its delivery commitments, during instances when their lead times exceed the expectation due to issues experienced by upstream stages.

In supply chain management, these networks are referred to as multi-echelon supply chains. They consist of nodes and arcs which depict stages and sequential relationships respectively, and the 'number of echelons' is defined as the largest number of nodes encountered along a path between the most upstream and most downstream of nodes. Depending on its node-arc configuration, multi-echelon supply chains are mainly classified as serial, assembly, distribution, general acyclic, general cyclic or spanning tree. Furthermore, allocating safety stock throughout the system so that target customer service times are achieved at the lowest cost is known as the multi-echelon inventory problem. Traditionally, this problem was approached using single-echelon inventory theory [26] which considered each stage independently. However, due to ignoring the many interdependencies in such a system, this method is clearly not optimal at most times. In the last decade, extensive research has been done on considering the multi-echelon inventory problem as requiring one large network solution, incorporating all interdependencies. The two main approaches taken in this regard are stochastic-service model (SSM) and guaranteed-service model (GSM).

In this paper, we will be considering the GSM approach, which (in its basic form) assumes that each stage has a deterministic service time it could guarantee the stage(s) in its downstream. It has been extensively studied in recent times under theoretical and industrial settings. Most notably GSM has been applied in many industrial applications including Eastman Kodak [7], Hewlett-Packard [1], P&G [5] and Intel [29], achieving significant reductions in inventory levels and holding costs. A comprehensive synthesis of related literature is provided in the next section of this paper.

Maynard [18] notes that building complex models that are ultimately intractable or developing highly efficient solution procedures, may run contrary to the practical nature of operations research (O.R). He goes on to say that this is mainly due to many analysts being poor at communicating the results of an O.R project in terms that can be understood and appreciated by practitioners who may not necessarily have a deep level of mathematical sophistication or formal training in O.R. Due to the complicated nature of the real-world networks, the substantial number of inputs and outputs and high computation times make GSM a somewhat complicated approach too. It is a particularly challenging tool in the context of day-to-day managerial decision making; which will involve evaluation of different options, what-if-analysis and scenario planning focused on just one or few stages. However, as noted previously, the real-world savings achieved via the GSM approach are significant and hence makes GSM a tool that cannot be ignored. Therefore, we believe that forming a procedure which lets GSM to be applied to multi-echelon supply chains in a more tractable manner and is less intensive on practitioners using it, is an important contribution.

We accordingly propose a procedure which enables GSM to be used in a convenient way for day-to-day decision making in the context of general acyclic multi-echelon supply chains [4] [5], which is a common configuration found among complex real-world multi-echelon supply chains. The procedure is useful in situations where quick, robust answers are sufficient during the initial decision-making process based on just a few variable changes; in other words, for cases where thousands of inputs and large computation times are not a viable option. However, following the initial decision-making stage, organizations may finalize their safety stock decision by doing a comprehensive GSM run and achieving optimal solutions.

To achieve this, we employ predictive global sensitivity analysis (PGSA) proposed by Wagner [28]. It lets us create structural equations based on known optimal results, using regression techniques. However, these equations will not necessarily require the traditional GSM inputs. Instead more understandable independent variables to regress for near-optimal solutions – which suffices for many cases of initial decision-making. These may be functional forms of exact GSM input variables, or latent variables which yields the same implications as more complicated functional forms do.

Our proposed methodology starts by generating the safety stock levels across the system on a periodic basis – daily, weekly, monthly or even seasonally – depending on how time sensitive the GSM inputs are. This will be done by formulating the problem as a concave cost mixed integer programming problem, and using successive piecewise linear approximations to obtain tight approximations to the general concave cost function, as proposed by Magnanti et al. [17]. This method provides optimal solutions for moderate sized supply chains, while near optimality for large supply chains. Following this, PGSA is used to obtain the regression-based structural equations; which can be used for day-to-day initial decision-making: evaluation of different options, what-if-analysis and scenario planning. In such decision making, it is highly unlikely that all nodes will required be adjusted. Therefore, the structural equations provide a convenient platform for evaluating the overall effect of input variations in one or few stages. The procedure allows its users to finalize safety stock levels to optimality, by running the procedure by Magnanti et al. [17] on the chain (if they opt to do so).

The remainder of the paper is organized as follows. In the next section, we review the literature related to this paper. We then introduce our optimization and PGSA model setups. Furthermore, we present computational experiments and their results, before outlining our conclusions and discussing future research directions.

RELATED LITERATURE

In this study, we draw mainly from three areas of existing work: (i) the guaranteed-service model (GSM) and its role in optimizing multi-echelon inventory systems, (ii) proposed solution methods for the GSM in theoretical and industrial contexts (iii) industry applications of the GSM and (iv) predictive global analysis.

The Guaranteed-Service Model (GSM)

The first area of interest (GSM) is one of many techniques proposed to optimize multi-echelon supply chains by achieving the objective of minimizing the total inventory holding cost and fulfilling set target service levels to the customers, by allocating safety stock across the supply chain. First proposed by Kimball [12] in a paper written in 1955 for a single-stage inventory system, the GSM was first applied to a multi-echelon inventory system by Simpson, Jr. [27] who considered a serial system. Current scholarly work consists of many complex applications of the GSM in multi-echelon inventory systems by improving upon the original assumptions (on the nature of external demand, lead times, applicable capacity constraints, service times, replenishment policies, etc.), mathematical formulations, the objective function, solution methods and by considering different types of supply chain networks (serial, assembly, distribution, general acyclic, general cyclic, spanning tree, etc.) as chronicled by Eruguz et al. [4]. In its general form, the GSM assumes deterministic service times at each stage in the supply chain and assigns safety stock levels to cover the net replenishment time, that is the difference (if any) in time between the stage receiving inventory from its upstream stages and the guaranteed delivery time guaranteed to its downstream customer. The stochastic-service model (SSM) is the other main approach presented by researchers in addressing the issue of optimizing safety stock levels, but the scope of this paper will be limited to the study and application of the GSM. Interested readers may refer survey papers by Diks, De Kok and Lagodimos [3], and Pianosi and Wagener [23] as well as literature comparing GSM and SSM approaches [8].

Solution Methods for the GSM

Although the GSM was introduced over 60 years ago, Eruguz et al. [4] note that 80% of the existing literature on the topic has been published over the last decade. Among these, we are most interested in scholarly work presenting solution methods for the resultant optimization problem, which is typically a minimization of a concave function over a closed, bounded convex set.

The state of optimality for the said optimization problem is explained by a property referred to in the literature as the ‘extreme point property’ [27]. This is a state where a stage either has sufficiently large safety stocks (which effectively disconnects it from downstream issues) or where a stage maintains no safety stocks. Numerous papers present optimality conditions based on this state (also referred to as an ‘all-or-nothing’ state), under different conditions and for different types of multi-echelon supply chains [10] [11] [15] [20]. As stated previously, we will focus on general acyclic networks, for which optimality properties were first presented by Minner [19]. Following the definition of these properties, a multitude of literature is available proposing dynamic programming approaches to solve the problem in the context of different supply chain types. In line with this study’s scope, we will emphasize the scholarly work done on general acyclic networks.

The proof of the NP-hard nature of the general acyclic network problem was presented by Lesnaia et al. [15], who also extend their work by proposing a branch-and-bound algorithm to obtain optimal solutions. We are particularly interested in the MIP based approach proposed to the general acyclic network problem by Magnanti et al. [17]. In proposing a novel successive piecewise linear approximation approach to the problem, Magnanti et al. [17] show that the problem can be solved, technically, as a sequence of MIP problems. Although this proves to be highly time consuming in the context of general acyclic networks, the authors also propose adding a set of redundant constraints to the formulation that refine the piecewise linear approximations. This method is able to solve moderate sized problems (approx.100 stages) using CPLEX, to optimality.

Many heuristic approaches have been subsequently proposed in order to overcome practical drawbacks of the procedure proposed by Magnanti et al. [17]. They mainly attempt to address its high computational requirements when used on larger networks [25] [16] [6].

Industry Applications

Eruguz et al. [4] classify the literature presenting industry applications of the GSM under two main categories: (i) papers that discuss the application of the GSM or an extension of it in a specific company, to improve its multi-echelon inventory management and, (ii) papers that consider the GSM and another managerial decision jointly (e.g. production planning, dual-sourcing, carbon emissions control, etc.).

While most of the industry applications of GSM are based on heuristics, the key results coming out of each application are largely favorable. Applying GSM techniques to optimize Intel’s complicated multi-echelon inventory system saw profound improvement in achieving the corresponding targets, as inventory levels dropped 11% while delivering service levels over 90% of the time [29]. Similarly Neale and Willems [21] discuss how the same techniques were effectively used in Microsoft at a time when the company had major concerns about the non-stationary stochastic demands experienced by their hardware business. Successful incorporation of a GSM resulted in an 18-20% increase in inventory turn, along with a 6-7% fill rate increase. Numerous similar studies and results are found in the literature across many industries from Hewlett-Packard [1] and Eastman Kodak [7] to Proctor & Gamble [5].

Despite all these being results obtained by applying different formulations of the GSM in many industry applications, we believe that improvements could be made to ensure that obtaining optimal/near optimal results is achieved through practical, convenient and sustainable. This is because, as it is, regularly running a system of GSM heuristics on a large multi-echelon system may entail many practical challenges: (i) it would require a large number of inputs (some of which may not be obviously available nor understandable, (ii) similarly the results following each run will contain large amounts of data which may not be easily interpretable, and (iii). the highly technical nature of the input and output details may devoid decision makers of any meaningful what-if-analysis or dynamic decision making – which are essential elements of any industrial setup.

Therefore to expand the practicality of the decision-making process associated with GSM's industrial applications, we propose a common-sense approach to applying GSM which is both accurate in its core task of optimizing the safety stock, holding costs and service levels, and sustainable to be used by managerial decision makers under a practical industrial setting. We will employ predictive global sensitivity analysis (PGSA) to develop this approach.

Predictive Global Sensitivity Analysis in GSM

In its most basic level, sensitivity analysis could be referred to as "the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input" [24, p. 45]. In the context of multi-echelon supply chains and so many other instances where decision making is based on optimization models, the sensitivity of the optimal output on varying parameter values plays an important role. However it becomes a meaningful practice only when the parameters and outputs bear interpretable meaning with respect to the decision being. In other words, when we have many parameters, the analysis becomes complicated and we need to go deep into the concepts of sensitivity, to handle this type of complexity [28].

This goes hand in hand with the fact that individual managers may not have the related software expertise, statistical and mathematical skills and therein won't be willing to devote the time to interpret and adjust such complicated models in meaningful ways. However, sensitivity-based approaches have the potential of addressing this issue directly.

Due to these reasons, sensitivity-based approaches have been increasingly proposed and applied in the field of operations research. Accordingly, Borgonovo and Peccati [2] implemented global sensitivity analysis techniques on an inventory management model previously proposed by one of the authors, which helped them solve a temporary sale issue in the former model. Hall and Posner [9] were the among first to apply sensitivity analysis on a scheduling problem, due to the environmental changes affecting these types of problems. The application of predictive global sensitivity analysis by Kouvelis et al. [13] in providing a conceptual framework that describes a firm's network structure along three dimensions: market focus, plant focus, and network dispersion. Lee and Munson's [14] also used the same techniques in proposing a set of structural equations to monitor the hedging positions over time and signal the need for potential change.

This study is meant to devise a fast and accurate approximation of the optimal solution given by GSM to the multi-echelon inventory problem, by utilizing the technique of predictive global sensitivity analysis. While working within Wagner's [28] global sensitivity analysis framework by carrying out *one-at-a-time regressions*, a polynomial combination of potential variables that managers may think as driving the decisions will be created to capture as much information as possible.

In summary, we contribute to the extant literature by integrating several of the above reviewed isolated streams of research. To the best of our knowledge, this is the first extensive study applying predictive global sensitivity analysis techniques in the GSM approach of multi-echelon inventory management. This will also be among the first studies of GSM, where simpler, more intuitive and convenient independent and dependent variables will be used in the form of robust structural equations, to obtain optimal or near optimal solutions.

MULTI-ECHELON SAFETY STOCK OPTIMIZATION MODEL

We consider the issue of multi-echelon inventory optimization, where a firm's goal is to find an optimal combination of service delivery times (or equivalently corresponding safety stock levels) across the firm's multi-echelon inventory system that minimizes the total inventory holding costs. The solution methods proposed in the literature for this problem involve complicated mathematical models and techniques, which may make day-to-day managerial decision making using these techniques less practical. We construct a set of structural equations by implementing a predictive global sensitivity analysis procedure to assist with day-to-day managerial decision making on multi-echelon inventory optimization.

Model Formulation

The most common type of multi-echelon inventory network configuration is the general acyclic network, which connects many stages of a supply chain system with no directed cycles. Due to reasons outlined in the literature review, we use the mathematical formulation by Graves and Willems [7] as adapted by Magnanti et al. [17].

Indices

j : Stage index, $\mathrm{j} = 1, \dots, \mathrm{N}$

k : Echelon index, $\mathrm{k} = 1, \dots, \mathrm{N_E}$

Sets

G : Acylic supply chain network

$\mathrm{A(G)}$: Arc set for the network representing the supply chain

$\mathrm{ECH(j)}$: The set of stages in the same echelon as stage j

SINK : The set of stages corresponding to finished goods (i.e. sinks in G)

SOURCE : The set of stages corresponding to suppliers (i.e. sources in G)

$\mathrm{DOWN(j)}$: The set of stages downstream of stage j

$\mathrm{DOWNIM(j)}$: The set of stages immedately downstream echelon of stage j

$\mathrm{DOWNSINK(j)}$: The set of sinks downstream of stage j

$\mathrm{UP(j)}$: The set of stages upstream of stage j

$\mathrm{UPIM(j)}$: The set of stages immedately upstream echelon of stage j

$\mathrm{UPSOURCE(j)}$: The set of sources upstream of stage j

Parameters

$\mathrm{T_j}$: Deterministic production (or assembly) lead time at stage j

$\mathrm{h_j}$: Per-unit holding cost for inventory at stage j

$\overline{\mathrm{S}}_\mathrm{j}$: Guaranteed service time to end customers at sink j

N : Number of possible stages in the network

$\mathrm{N_E}$: Number of echelons in the network

Decision Variables

S_j : Guaranteed service time each stage j promises to its downstream customers

SI_j : An upperbound of form (i) $SI_j \geq \max\{S_i | i: (i,j) \in A(G)\}$ and (ii) $SI_j \geq S_j - T_j$

Functions

Φ_j : The level of safety stock required, as a function of the days of demand required

$$\text{minimize} \quad \sum_{j=1}^{N} h_j \, \Phi_j\big(SI_j + T_j - S_j\big) \tag{1}$$

subject to:

$$S_j - SI_j \leq T_j \quad \forall\, j \tag{2}$$
$$SI_j - S_i \geq 0 \quad \forall\, (i,j) \in A(G) \tag{3}$$
$$S_j \leq \bar{S}_j \quad \forall\, j \in SINK \tag{4}$$
$$S_j, SI_j \geq 0 \quad \forall\, j \tag{5}$$

The objective function (1) minimizes the total holding cost across the system, where Φ_j is a concave and non-decreasing function denoting the level of safety stock required, as a function of the days of demand required. Constraint (2) ensures a non-negative net replenishment time at each stage, while constraint (3) ensures that the inbound service time of a stage is no smaller than the service times of its upstream suppliers. Constraint (4) ensures that all final (demand) stages satisfy their service guarantees to the customers [17]. Finally, constraint (5) enforces the non-negativity of the decision variables.

We assume in this model that the production operation is deterministic, but normally distributed customer requirements may be experienced at sinks. The system also needs to meet specified service time guarantees to deliver the finished goods to consumers. It is also assumed that each node operates under a periodic-review and base-stock policy and that each downstream stage is assumed to require one unit of any (connected) adjacent upstream stage. We also assume that transportation times between the stages are zero and that each stage always meets the service times in which it promises to make the delivery.

PREDICTIVE GLOBAL SENSITIVITY ANALYSIS

As described in previous sections, we will now construct structural equations for day-to-day decision making using a predictive global sensitivity analysis procedure. This process will be initiated by generating a dataset consisting of a variety of input parameters and the corresponding solutions obtained by using the deterministic model introduced previously.

Model Formulation

We run regressions on the dataset created, to obtain the structural equations. The dependent variables in this regression process are model decision variables whereas independent variables are summary measures defined as combinations of the input parameters. The independent variables are defined such that they are simple enough for managers to be able to use in day-to-day decisions, yet are capable of capturing the complexity and details of the original mathematical program [14]. As outlined by Kouvelis, Munson, & Yang, [13], separate *one-time regressions* [28] are used to determine which of the variables should be included in the final model. This is done by attempting a polynomial fit for $x^{\Delta} \in$ {-6, -5, …, -1, 1, …, 5, 6}. This provides a measure of each independent variable's effect on each of the dependent variables, as the independent variables with the highest R^2 are considered to possess the highest influence.

We next combine these influential independent variables to form one regression. As part of this process, we account for interaction effects and nonlinearities by combining each pair of independent variables in the form $x^{\Delta 1}y^{\Delta 2}$ with $\Delta 1, \Delta 2 \in$ {-2, -1, 1, 2}. Similarly, in the case of 3 independent variables, the combination is done as follows: $x^{\Delta 1}y^{\Delta 2}z^{\Delta 3}$ with $\Delta 1 \in$ {-2, -1, 1, 2}, $\Delta 2 \in \{\pm \Delta 1\}$, and $\Delta 3 \in \{\pm \Delta 2\}$. Stepwise regression is then used to eliminate and keep said terms to finally form structural equations for each of the two dependent variables. SAS® is used to execute this, setting a α of 0.05 to enter and 0.10 to remove independent variables from the final structural equations [14] [13]. The expectation is for these equations to exhibit high Adjusted R^2 values for each of the remaining independent variable terms.

Independent Variables

We present four types of independent variables to be analyzed. These variables have been defined considering the practical scenarios that managers will face, in which a predictive global sensitivity analysis would be required. Managerial decision making with respect to a multi-echelon inventory system on a day-to-day basis is most likely to involve just one or few stages in the network, based on operational issues. Therefore, the variables (6) - (19) are defined to capture the conditions faced by a given stage j, which will most likely have a strong effect on the guaranteed service levels that would be assigned to j.

$$\text{Location Relative Holding Cost: LOCHOLD} = \frac{\frac{\sum_{j \in DOWN(j)} h_j}{|DOWN(j)|}}{\frac{\sum_{j \in UP(j)} h_j}{|UP(j)|}} \tag{6}$$

$$\text{Downstream Relative Holding Cost: DRHOLD} = \frac{h_j}{\frac{\sum_{j \in DOWN(j)} h_j}{|DOWN(j)|}} \tag{7}$$

$$\text{Immediate Downstream Relative Holding Cost: IDRHOLD} = \frac{h_j}{\frac{\sum_{j \in DOWNIM(j)} h_j}{|DOWNIM(j)|}} \tag{8}$$

$$\text{Upstream Relative Holding Cost: URHOLD} = \frac{h_j}{\frac{\sum_{j \in UP(j)} h_j}{|UP(j)|}} \tag{9}$$

$$\text{Immediate Upstream Relative Holding Cost: IURHOLD} = \frac{h_j}{\frac{\sum_{j \in UPIM(j)} h_j}{|UPIM(j)|}} \tag{10}$$

$$\text{Echelon Relative Holding Cost:} \quad \text{ECHOLD} = \frac{h_j}{\frac{\sum_{j \in ECH(j)} h_j}{|ECH(j)|}} \tag{11}$$

$$\text{Relative Distance from Sink:} \quad \text{RSINK} = \frac{|DOWNSINK(j)|}{|DOWN(j)|} \tag{12}$$

$$\text{Relevant Sink Proportion:} \quad \text{SINKPROP} = \frac{|DOWNSINK(j)|}{|SINK|} \tag{13}$$

$$\text{Relative Distance from Source:} \quad \text{RSOURCE} = \frac{|UPSOURCE(j)|}{|UP(j)|} \tag{14}$$

$$\text{Relevant Source Proportion:} \quad \text{SOURCEPROP} = \frac{|UPSOURCE(j)|}{|SOURCE|} \tag{15}$$

$$\text{Approx. Stock Day Requirement: ASDR} = \frac{\sum_{j \in DOWN(j)} T_j - \sum_{j \in DOWNSINK(j)} \bar{S}_j}{|DOWNSINK(j)|} + T_j \tag{16}$$

$$\text{Service Requirement:} \quad \text{SERVEREQ} = \frac{\sum_{j \in DOWNSINK(j)} \bar{S}_j}{\sum_{j \in SINK(j)} \bar{S}_j} \tag{17}$$

$$\text{Echelon of the Stage: ECHELON} = k \tag{18}$$

$$\text{Relative Echelon of the Stage: RELECH} = \frac{k}{N_E} \tag{19}$$

Dependent Variables

The decision variables in the model we use are limited (just two values: S_j and SI_j), and the major managerial decisions expected to be made based on a similar scenario are directly about the service time guarantees that a stage could make to its downstream customer. Therefore, the decision variables S_j and SI_j will directly be used as the dependent variables in forming two structural equations.

COMPUTATIONAL EXPERIMENTS

In this section we describe the computational process used and results obtained for the presented model setup. We used MATLAB to run the algorithm and AMPL to solve the non-linear optimization problem for minimizing system wide inventory holding costs.

Network Definition

A binary adjacency matrix ($N \times N$ square) models the multi-echelon network, which could have up to N stages and N_E echelons as specified. Non-zero elements of the adjacency matrix represent active arcs between nodes, and we assign them in each case based on defining characteristics of the multi-echelon inventory systems described by Farasyn et al. [5], Wieland et al. [29] and Magnanti et al. [17].

All stages forming the first echelon (sources) consist of all possible outgoing arcs, connecting them to each of their immediate downstream stages. However, from the next echelon onwards we progressively decrease the probability of an outgoing arc being assigned to a stage, to mimic the real-life observation of decreasing arc densities as stages get closer to their sinks. We enforce this by setting the (downstream) arc-assignment probability of a stage to $(N_E - ECH_i)/(N_E - 1)$. Furthermore, we assume that an arc could only connect stages located in immediately neighboring echelons.

Following this assignment of arcs, we conduct checks and corrections (if needed) to ensure continuity among any stages with arcs coming into or out of them. That is, to ensure that any non-sink stages with incoming arcs would also contain outgoing arcs.

Parameter Generation

We follow the procedure used by Magnanti et al. [17] to generate values for stage parameters $T_j, \overline{S_j}$, and h_j. Accordingly, T_j follows the distribution $U[1,100]$ and $\overline{S_j}$ follows the distribution $U[1,10]$. Moreover, we set $h_j = h_i + U[1, 100]$, where h_i is the largest holding cost rate of its immediate upstream stages. This ensures that T_j and $\overline{S_j}$ values distribute uniformly for all relevant stages, whereas h_j values reflect the fact that inventory holding is generally costlier as the stages move downstream. We define the concave and non-decreasing function at each stage as $\Phi_{\mathrm{j}}(X) = X$.

We perform optimization procedures using the MINOS non-linear solver on AMPL.

One-at-a-time Regression

One-at-a-time regressions [28] determine which of the 14 independent variables should be carried forward toward forming the final multiple linear regression model. We execute this selection by comparing the adjusted R^2 values calculated by attempting a polynomial fit for $x^{\Delta} \in$ {-6, -5, …, -1, 1, …,6}, for each independent variable. We deem the independent variables corresponding to the top three adjusted R^2 values to be potentially possessing the highest influence.

TABLE 1. Most Influential Independent for each Network Setup Considered

Number of Nodes	Number of Echelons		
	3	4	5
25 to 50	*LOCHOLD, SINKPROP, SERVREQ*	*LOCHOLD, RSOURCE, RSINK*	*LOCHOLD, DRHOLD, RSOURCE*
50 to 100	*LOCHOLD, URHOLD, ASDR*	*LOCHOLD, URHOLD, RSOURCE*	*LOCHOLD, RSINK, RSOURCE*

100 to 200	*LOCHOLD, URHOLD, IURHOLD*	*LOCHOLD, RSINK, RELECH*	*LOCHOLD, RSINK, RSOURCE*

To provide a comprehensive analysis of the most influential independent variables, the experimental design considers nine different multi-echelon network configurations. We achieve this variety of configurations by varying both the number of stages (nodes) in a network (e.g.: 25 to 50 stages, 50 to 100 stages, 100 to 200 stages), and the number of echelons in a network (e.g.: 3, 4 and 5 echelons). Table 1 outlines the most influential independent variables (based on R^2), for each network configuration considered.

Formation of Structural Equations

Following the one-at-a-time regression, we combine the top three independent variables corresponding to each network to form 16 terms as follows: $x^{\Delta 1} y^{\Delta 2} z^{\Delta 3}$ with $\Delta 1 \in$ {-2, -1, 1, 2}, $\Delta 2 \in \{\pm \Delta 1\}$, and $\Delta 3 \in \{\pm \Delta 2\}$. Using MATLAB, we then execute stepwise regression setting α-values of 0.05 to enter and 0.10 to remove independent variables. Finally, any terms retained through this process form the relevant structural equations for each of the nine network configurations.

For the example configuration of $\mathrm{N} = 25 \text{ to } 50$; $\mathrm{N_E} = 3$, the results are as follows:

• The independent variables yielding the top three adjusted R^2 are,

$$\text{LOCHOLD} = \frac{\frac{\sum_{j \in DOWN(j)} h_j}{|DOWN(j)|}}{\frac{\sum_{j \in UP(j)} h_j}{|UP(j)|}} \qquad \text{SINKPROP} = \frac{|DOWNSINK(j)|}{|SINK|} \qquad \text{SERVEREQ} = \frac{\sum_{j \in DOWNSINK(j)} \overline{S_J}}{\sum_{j \in SINK(j)} \overline{S_J}}$$

• Following the stepwise regression of the 16 different configurations of LOCHOLD, SINKPROP, and SERVEREQ, the final structural equation formed using the retained combinations is,

$$s_J = 5.4737 - 303.29\left(\frac{SINKPROP \times SERVEREQ}{LOCHOLD}\right) + 3132.8\left(\frac{SINKPROP \times SERVEREQ}{LOCHOLD}\right)^2$$

• This structural equation results in an adjusted R^2 of 0.896.

Table 2 tabulates the adjusted R^2 values corresponding to the final structural equations for each of the nine network configurations considered.

TABLE 2. Adjusted R^2 Values for the Final Structural Equations of each Network Configuration Considered

Number of Nodes	**Number of Echelons**		
	3	**4**	**5**
25 to 50	*0.896*	*0.657*	*0.660*
50 to 100	*0.783*	*0.758*	*0.832*
100 to 200	*0.702*	*0.713*	*0.924*

Model Validation

We then validate the final structural equations (for each configuration) by calculating the mean absolute deviation (MAD), mean squared error (MSE), and MAD/Mean ratio for different networks fitting the same configuration. The purpose of calculating the MAD/Mean ratio was as an alternative to the more widely used mean average percent error (MAPE) due to the large number of zero values in each dataset, which does not warrant the use of MAPE.

For the example configuration of $\mathrm{N} = 25 \text{ to } 50\,;\, \mathrm{N_E} = 3$, the model validation results are as follows:

- Mean absolution deviation (MAD) $= \frac{\sum_N |S_j - Regressed\ Value_j|}{N}$

  $= 26.1659$

- Mean Squared Error (MSE) $= \frac{\sum_N (S_j - Regressed\ Value_j)^2}{N}$

  $= 1.6059 \times 10^3$

- MAD / Mean $= \left(\frac{\sum_N |S_j - Regressed\ Value_j|}{\sum_N (S_j)}\right)$

  $= 0.7847$

Table 3 tabulates the $(\mathrm{MAD/Mean})$ values given by the final structural equations for each of the nine network configurations considered.

TABLE 3. $(\mathbf{MAD/Mean})$ Ratios for the Final Structural Equations of each Network Configuration Considered

Number of Nodes	Number of Echelons		
	3	4	5
25 to 50	*0.78*	*4.29*	*4.66*
50 to 100	*1.50*	*2.70*	*220.86*
100 to 200	*3125*	*2142*	*5.68*

CONCLUSIONS

Following the one-at-at-time regressions of the PGSA procedure, we developed and tested independent variables which individually correlate with the optimal results obtained. We formed these independent variables for each stage in multi-echelon networks, using managerially convenient inputs which capture different characteristics of each stage.

Furthermore, we used these variables to form structural equations which demonstrated reasonably large R^2 with the optimal results. We observed that the structure and independent variables included in these equations vary with the number of stages in a network and the number of echelons in a network. This underscores the importance of the proposed procedure as a tool for initial decision making, as it captures differences in the characteristics of networks when producing near-optimal estimates.

Conventional error measurements such as MAD, MSE and (MAD/Mean) are relatively high, implying that the PGSA models do not successfully address the objectives of this project. However, we further observed that these measurements almost exclusively increase with the size of the problem considered.

This is suggestive that the structural characteristic of the optimal solutions of resulting in many zero-values (i.e. no safety stock holding), is inflating these conventional error measurements. Accordingly, we also conclude that since the PGSA approach considered results in a general regression equation for each network, it is highly unlikely that the current approach would result in regressed values which successfully replicate this zero-value characteristic of the optimal results. In other words, we note that direct application of the conventional PGSA approach may be insufficient for the purpose of making predictions on optimal safety stock levels of multi-echelon networks.

PROPOSED EXTENSIONS

As we concluded the structure of the optimal results is suggestive of direct application of a regression approach such as the conventional PGSA approach being insufficient, further work needs to be done to develop a more adaptable regression approach. The initially high R^2 values suggest that the PGSA approach is successful under the right conditions; especially when the number of zeros in the optimal results is smaller. This leads us to believe that a heuristic-based approach may need to be combined with the conventional PGSA approach to better mimic the structure of optimal solutions.

Furthermore, additional measures beyond the conventional error measurement approaches will need to be developed to evaluate the performance of such a model, which overcomes the challenges posed by the structure of the optimal solutions.

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