A SPECIAL MODEL FOR DETERMINING EXPECTED NUMBER OF FUTURE FAILURES

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ABSTRACT

In order to allocate resources to perform future repairments of an object, the expected number of future operational failures have to be determined. This paper offers a model and solution algorithm in the case of a special type of repairs. The model is a special integral equation and for its solution a numerical integration-based algorithm is proposed. A simple numerical example illustrates the methodology.

Keywords: Expected Failures, Reliability, Simulation, Modeling

INTRODUCTION

Reliability engineering is a very important engineering field which applies scientific methodology to ensure that a system, process, equipment, or part performs as desired. It includes the analysis and prediction of failures which occur usually randomly. The times to failures are therefore characterized as random variables and the methodology of probability theory is the usual tool of analysis and prediction. There is a large amount of literature discussing the fundamentals of reliability engineering (Barlow and Proschan [1]; Jardine and Tsang [2]; Nakagawa [3], among others).

If failure occurs, then repairment is needed. Three categories of repairment are considered. Minimal repairs restore the object to the state it had just before failure. Replacement means that the object is replaced by a new one. Partial repair results in a state which is better than that before failure occurred. The effective age of an object is τ at time t when the degradation of the object in interval $[0, t]$ is the same as it would have been working on the entire interval $[0, \tau]$.

Partial repairs might decrease the effective age by a constant term, or by a certain percentage. It is also considered often that partial repair decreases the expected number of future failures with some percentage meaning that its expectation is multiplied by a factor less than unity. In this paper this type of partial repairs are considered, the others can be analyzed in a similar way.

In planning future repairs, allocating parts, tools, and skilled workforce it is very important to estimate the expected number of failures in any future time intervals and the structure of their appearances. The mathematical methodology dealing with these issues is based on the renewal theory, the fundamentals of which are presented, for example, in Cox [4] and Ross [5]. The expected numbers of future failures are discussed in special cases in Goodman et al. [6] assuming different types of repairs ignoring degradation of idle object, which cannot be ignored by the environmental effects such as temperature, air humidity, etc. In this paper this effect will be also incorporated into the analysis.

This paper is organized as follows. Section 2 contains the main mathematical development and a solution algorithm. Section 3 offers a numerical example illustrating the methodology and Section 4 presents final comments and future research directions.

THE MATHEMATICAL MODEL AND SOLUTION

Consider an equipment which is subject to random failures. Let $F(t)$ denote the CDF of the first failure with pdf $f(t)$. After a failure occurs a repair takes T time units in which the equipment's effective age increases by dT, where $d < 1$ is a given constant. It is also assumed that repairs do not slow down degradation but lowers its consequence by decreasing the number of future failures by a factor $\alpha < 1$ with known α value. It is very important to estimate the expected number of failures in any time interval $[0, t]$, which is denoted by $M\ (t)$. We can use the well-known idea of expectation by conditioning. Let S_1denote the time of the first failure.

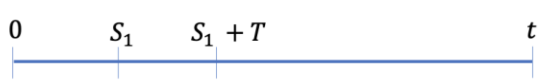

Figure 1.

The effective age of the equipment at time S_1 is S_1, at time $S_1 + T$ it is $S_1 + dT$, and the final time t it becomes $t - T + dT$ if $t > S_1 + T$. Note that between S_1 and $S_1 + T$ the degradation is dT instead of T.There is no failure in this interval. Let X_t denote the true number of failures in interval $[0, t]$, then:

$$E(X_t|S_1) = \begin{cases} 0 & if \;\; t < S_1 \\ 1 & if \;\; S_1 \le t < S_1 + T \\ 1 + \alpha[M(t - T + dT) - M(S_1 + dT)] & if \;\; t \ge S_1 + T \end{cases} \tag{1}$$

So

$$M(t) = \int_0^{t-T} \{1 + \alpha[M(t - T + dT) - M(s + dT)]\} f(s) d(s) + \int_{t-T}^{t} f(s) ds + \int_t^{\infty} 0 ds$$

$$= F(t) + \alpha M(t - T + dT) F(t - T) - \int_0^{t-T} \alpha M(s + dT) f(s) \, ds \tag{2}$$

if $t > T$. Otherwise, there is a failure with probability of $F(t)$ and no more failure could occur, since repairment cannot be finished before time t. So in this case:

$$M(t) = F(t) \qquad for \; t < T \tag{3}$$

This is an integral equation for the unknown function $M(t)$. The expected number of failures in any interval $[t_1, t_2]$ is clearly $M(t_2) - M(t_1)$, which has a huge practical importance to help to locate material needed for repair and also skilled manpower.

Equation (2) can be solved by using numerical integration, the right hand side depends on only the unknown function values between dT and $t - T + dT$ which are smaller than t. Between 0 and T function $M(t) = F(t)$ which is known. So taking $t = T + \hbar, T + 2\hbar, \ldots$ and using numerical integration based on nodes $0, \hbar, 2\hbar, ..$ on the right hand side the unknown function values can be obtained in the order of $M(T + \hbar), M(T + 2\hbar), M(T + 3\hbar), ..$

It is advised to select the small step size $\hbar$ such that dT is an integer multiple of $\hbar$, $dT = K\hbar$. Therefore $M(k\hbar) = F(k\hbar)$ for $k \le L$ where $L\hbar \le T < (L + 1)\hbar$. If $t = T + n\hbar$ $(n \ge 1)$, then

$$M(T + n\hbar) = F(T + n\hbar) + \alpha M((n + K)\hbar) F(n\hbar) - \alpha \hbar \sum_{\ell=0}^{n} A_{n,\ell} M((\ell + K)\hbar) f(\ell\hbar) \tag{4}$$

where $A_{n,\ell}$ as the quadrature coefficients. For example, using the trapezoidal rule, $A_{n0} = A_{nn} = \frac{1}{2}$ and, $A_{nk} = 1$ $(1 \le k \le n-1)$.

NUMERICAL EXAMPLE

Assume that the time to failure distribution is Weibull with CDF $F(t) = 1 - e^{-\left(\frac{t}{\lambda}\right)^k}$, and expectation $\lambda\Gamma(1 + \frac{1}{k})$. The parameters are selected as $\lambda = k = 2$. Let $\alpha = 0.9$ and $d = 0.1$ and $T = 1$. In the numerical integration we select $\hbar = 0.01$ and interval $[0, 10]$. In computing $M(10)$ algorithm (4) is used. Table 1 shows values of $M(t)$ for several earlier times as well. We can see that $M(t)$ increases in t as it should and finally $M(10)$ =7.89.

In our computation the trapezoidal rule was applied. The fundamentals of numerical integration with some useful formulas can be found, for example, in Yakowitz and Szidarovszky [7].

Table 1. Model Solution

$\boldsymbol{t}$	$\boldsymbol{M(t)}$
0	0
1	0.221
2	0.496
3	1.138
4	2.043
5	3.084
6	4.154
7	5.191
8	6.168
9	7.069
10	7.891

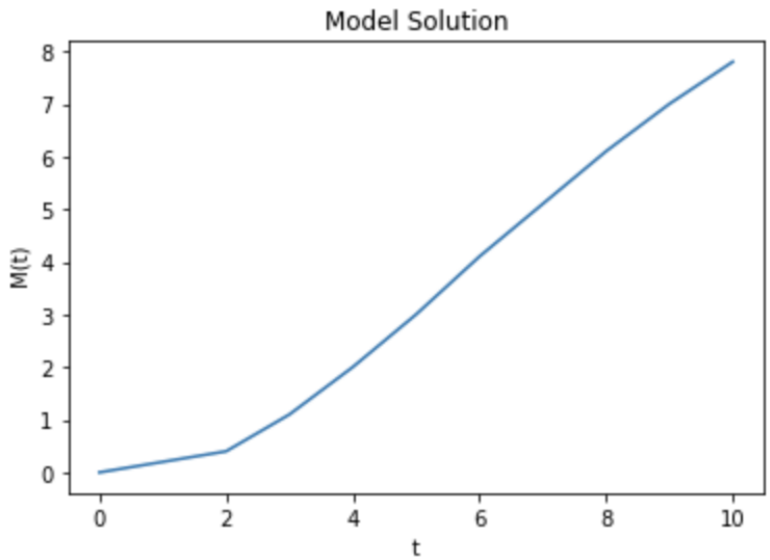

Figure 2. Model Solution Linear Plot

CONCLUSION

In order to maintain the desired operation of systems, processes, equipment and parts, it is important to analyze the structure and properties of repeated future failures. Their expectation in any future interval helps the management to allocate resources to guarantee fast repairs when failure occurs.

An integral equation was derived to the expected number $M\ (t)$ of failures in interval $[0, t]$ for any given $t\ >\ 0$. Based on this result, the expected number of failures in intervals $[t_1,\ t_2]$ can be obtained as $M\ (t_2) - M\ (t_1)$. Numerical integration-based solution algorithm was offered to solve the integral equation, and in the illustrating numerical example the trapezoidal rate was used.

It is an interesting question to assess the distribution of the actual number of failures in any interval $[t_1,\ t_2]$ to see the structure of future failures in more detail.

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