A TEACHING APPROACH FOR THE ECONOMIC PRODUCTION QUANTITY MODEL USING ONLY ALGEBRA AND ANALYTIC GEOMETRY

Cenk Çalışkan, Woodbury School of Business, Utah Valley University, 800 W. University Pkwy, Orem, UT 84058, (801) 863-6487, cenk.caliskan@uvu.edu

ABSTRACT

It is often a challenge to teach the Economic Production Quantity (EPQ) model to undergraduate students, because it requires the use of differential calculus, and many undergraduate business students are not prepared for such higher mathematics subjects. The challenge is especially pronounced when there are two decision variables, as in the backordering extension of the model. In this paper, we demonstrate an approach to teach the EPQ model with backordering by using only algebra and analytic geometry. The approach is applicable to any minimization or maximization problem where the objective function is continuously differentiable. Based on our experience in an operations management course, the students enjoy learning the subject using the proposed approach, learn better and retain the information longer. The proposed approach has great potential as a pedagogical tool in teaching inventory management to students with less mathematical backgrounds.

Keywords: inventory; EPQ; economic production quantity, algebraic methods, teaching, pedagogy.

INTRODUCTION

Since the development of the basic Economic Order Quantity (EOQ) model by Harris [12], there has been numerous extensions of the basic model to relax its somewhat restrictive assumptions and include more realism. One such extension is the Economic Production Quantity (EPQ) model, which relaxes the instantaneous inventory replenishment assumption and allows for gradual accumulation of the inventory over time. Harris [12] states that “the solution of this problem requires higher mathematics.” Many undergraduate business students tend to agree with him on this statement, and it is somewhat of a challenge to teach them the subject because of the use of differential calculus in optimizing the model. This challenge becomes more pronounced when a second decision variable is added to the model, such as the number of units to backorder in each production cycle. To simplify the derivation of optimal solutions, a number of non-calculus methods have been proposed in the literature for both EOQ and EPQ problems, but they are not generalizable to other optimization problems encountered in business and economics, and they are more complicated than the standard approach, therefore defeating the purpose. Another problem with the existing approaches is that they make some assumptions about the characteristics of the optimal solution, which cannot be known before optimizing the model. In this paper, we propose and demonstrate a simple approach to teach the EPQ problem with and without backorders that is based only on algebra and analytic geometry. We have some anecdotal experience from a core operations management course that indicates that the students enjoy learning the material more and retain the information longer, when this approach is used. It is also a good approach for practitioners to learn the model, who may lack working knowledge or background of differential calculus. Even though there is some algebraic manipulation in our approach that may look complicated at first look, compared to the algebraic approaches in the literature, it is much simpler and more intuitive.

LITERATURE REVIEW

Grubbström and Erdem [11] develop an algebraic approach to find the optimal solution to the EOQ problem. Cárdenas-Barrón [1] extends this approach to the EPQ problem. Wee et al. [21] extend the approach of Grubbström and Erdem [11] to the case where there is a temporary price change, and Huang [13] extend both EOQ and EPQ versions of the approach of Grubbström and Erdem [11] to imperfect items with backordering. Ronald et al. [16] criticize Grubbström and Erdem [11] for using the a priori information that the ordering cost (setup cost in the EPQ model) per unit time equals the sum of inventory holding and backordering costs per unit time at the optimal solution. It is true that the approach in Grubbström and Erdem [11] and its later extensions use this a priori information about the optimal solution, which makes it only a verification rather than a derivation, but Ronald et al. [16] also use an a priori information about the optimal solution: that the optimal backorder quantity is proportional to the optimal order quantity.

Another algebraic approach is developed by Sphicas [17], which is called "complete the perfect square" method. In this method, one adds and subtracts an extra term to the total cost equation to turn it into the square of an expression, for which the optimal solution is obvious and obtained by setting the expression to zero. Cárdenas-Barrón [2] extend this approach to the EPQ problem with rework. Huang et al. [14] extend it to the EPQ problem with a cash discount and permissible delay in payments.

Arithmetic mean of a set of nonnegative numbers is always greater than or equal to their geometric mean. Teng [19] uses this inequality to develop an algebraic solution approach for the EOQ and EPQ problems without backordering. Cárdenas-Barrón et al. [4] extends this approach to a two-echelon system which he calls a vendor-buyer system. The product of the sums of squares of any two sequences of real numbers is always greater than or equal to the square of the sum of products of the individual numbers in the two sequences in the same position, which is known as the Cauchy–Bunyakovsky–Schwarz inequality. Cárdenas-Barrón [3] uses the two inequalities to develop an approach to solve both EOQ and EPQ problems with backordering. Teng et al. [20] combines the complete the perfect square and the arithmetic mean-geometric mean inequality approaches of Sphicas [17] and Teng [19], respectively.

Based on a marginal analysis approach, Minner [15] develops yet another algebraic method which he calls "cost comparisons" to solve the EOQ and EPQ problems. This same approach is later adapted by Wee et al. [22] for optimizing the order quantity as opposed to the order interval. Chung [10] and Widyadana et al. [23] later adapt the cost comparisons method to vendor-buyer and deteriorating items extensions of the EOQ and EPQ models. Çalışkan [7] and Çalışkan [8] demonstrate that the cost comparisons approach is equivalent to using the first order conditions for optimality, and that the aforementioned papers do not check the second order conditions.

All of the mentioned approaches are extremely complicated, involving long algebraic derivations, which defeats the purpose of simplifying the mathematics for students who are not well-versed in calculus. We present a very simple, intuitive and short method to solve the EPQ problem with backorders that can be easily understood by anyone who knows some algebra and analytic geometry. It is a simplified adaptation of the approach that is applied to the deteriorating items inventory models in Çalışkan [5], Çalışkan [6] and Çalışkan [9]. Because we use only a modest level of algebra and analytic geometry, it is a great pedagogical tool to be used in the classroom to teach undergraduate business students.

THE BASIC ECONOMIC PRODUCTION QUANTITY MODEL

In the basic EPQ model, contrary to the basic EOQ model, inventory replenishment is not instantaneous. The replenishment happens at a constant rate over time, which is the production rate. The items are consumed by the demand. Production and consumption are simultaneous until the production output reaches the batch size; and from that time until the next batch is started, there is only consumption that depletes the inventory. The following are the variables and the parameters of the basic EPQ model:

D = the demand rate per unit time
P = the production rate per unit time
S = the cost of setup per batch
h = the cost of inventory holding per unit per unit time
Q = the number of units to produce in each production cycle (batch size)
T = the time between batch starts (cycle length)

Let $\rho = \frac{P-D}{P}$. It is well-known that the maximum inventory level I_{max} can be determined as follows (see, for instance, Stevenson [18]):

$$I_{max} = \frac{Q}{P}(P - D) = \rho Q \tag{1}$$

The average per unit time total cost can then be expressed as follows:

$$TC(Q) = \frac{SD}{Q} + h\rho\frac{Q}{2} \tag{2}$$

Fig. 1 shows the plot of $TC(Q)$ with respect to Q. Let Q^* be the optimal batch size that minimizes Eq. 2 and let

$$Q_u = Q^* + \Delta Q \tag{3}$$
$$Q_l = Q^* - \Delta Q \tag{4}$$

for some $\Delta Q > 0$. We can see in Fig. 1 that the optimal batch size Q^* satisfies the following inequality for any $Q_l > 0$:

$$TC(Q_l) - TC(Q^*) \geq 0 \tag{5}$$

We can also say based on Fig. 1 that the following will also hold:

$$TC(Q_u) - TC(Q^*) \geq 0 \tag{6}$$

Eq. 5 can further be simplified as follows:

$$\left[\frac{SD}{Q_l} + \frac{h\rho Q_l}{2} - \frac{SD}{Q^*} - \frac{h\rho Q^*}{2}\right] \geq 0$$
$$SD\frac{(Q^* - Q_l)}{Q_l Q^*} + \frac{h\rho}{2}(Q_l - Q^*) \geq 0$$

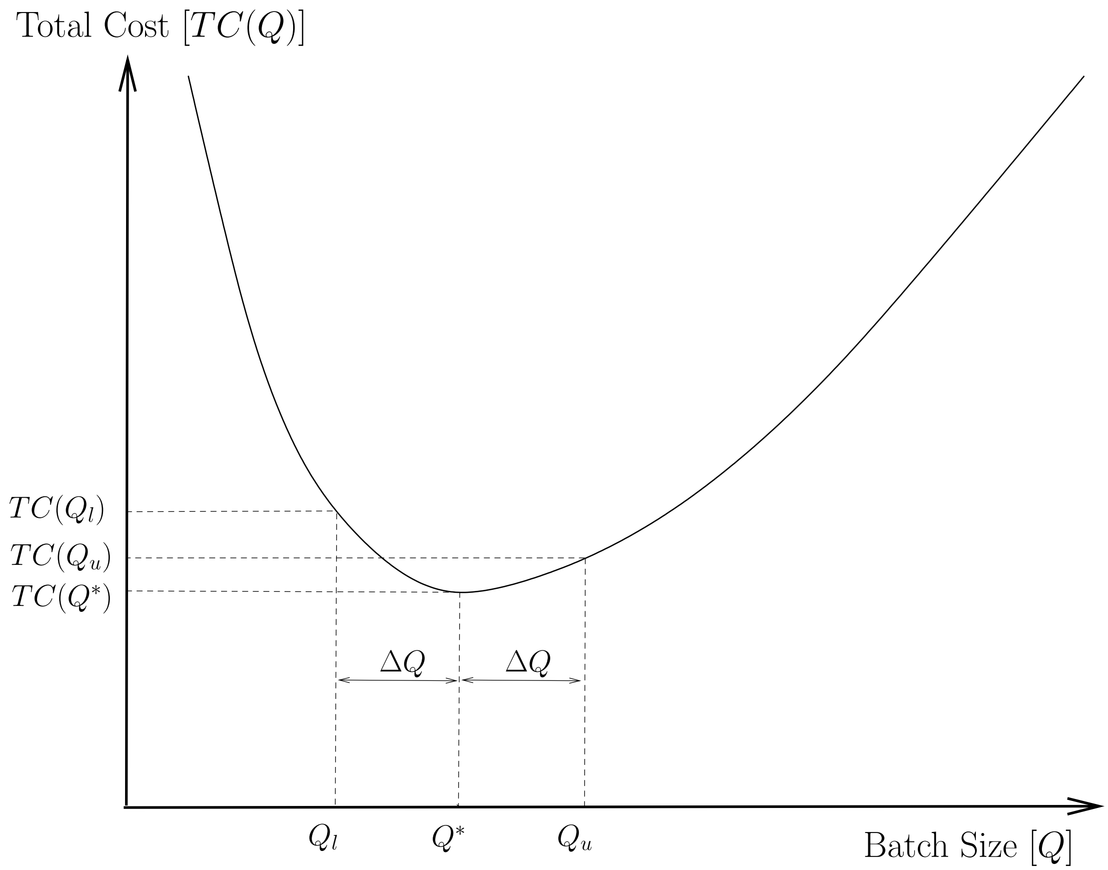

Figure 1: The total cost function $TC(Q)$ and the optimality of $Q = Q^*$

$$-\frac{SD}{Q_l Q^*} + \frac{h\rho}{2} \leq 0 \tag{7}$$

Similarly, Eq. 6 can be simplified as follows:

$$\left[\frac{SD}{Q_u} + \frac{h\rho Q_u}{2} - \frac{SD}{Q^*} - \frac{h\rho Q^*}{2}\right] \geq 0$$

$$SD\frac{(Q^* - Q_u)}{Q_u Q^*} + \frac{h\rho}{2}(Q_u - Q^*) \geq 0$$

$$-\frac{SD}{Q_u Q^*} + \frac{h\rho}{2} \geq 0 \tag{8}$$

Thus, Eqs. 7 and 8 result in the following inequality:

$$\left[-\frac{SD}{Q_u Q^*} + \frac{h\rho}{2}\right] \geq 0 \geq \left[-\frac{SD}{Q_l Q^*} + \frac{h\rho}{2}\right] \tag{9}$$

As we decrease ΔQ, approaching $\Delta Q = 0$, both Q_l and Q_u approach Q^*. Furthermore, the two sides of Eq. 9 approach each other. Therefore, when ΔQ is approaching zero, the following holds:

$$-\frac{SD}{(Q^*)^2} + \frac{h\rho}{2} = 0$$

$$Q^* = \sqrt{\frac{2DS}{h\rho}}$$

$$Q^* = \sqrt{\frac{2DS}{h}}\sqrt{\frac{P}{P-D}} \tag{10}$$

Thus, we have just determined the EPQ equation that gives us the batch size that minimizes the total cost.

Proving the Uniqueness of the Optimal Solution

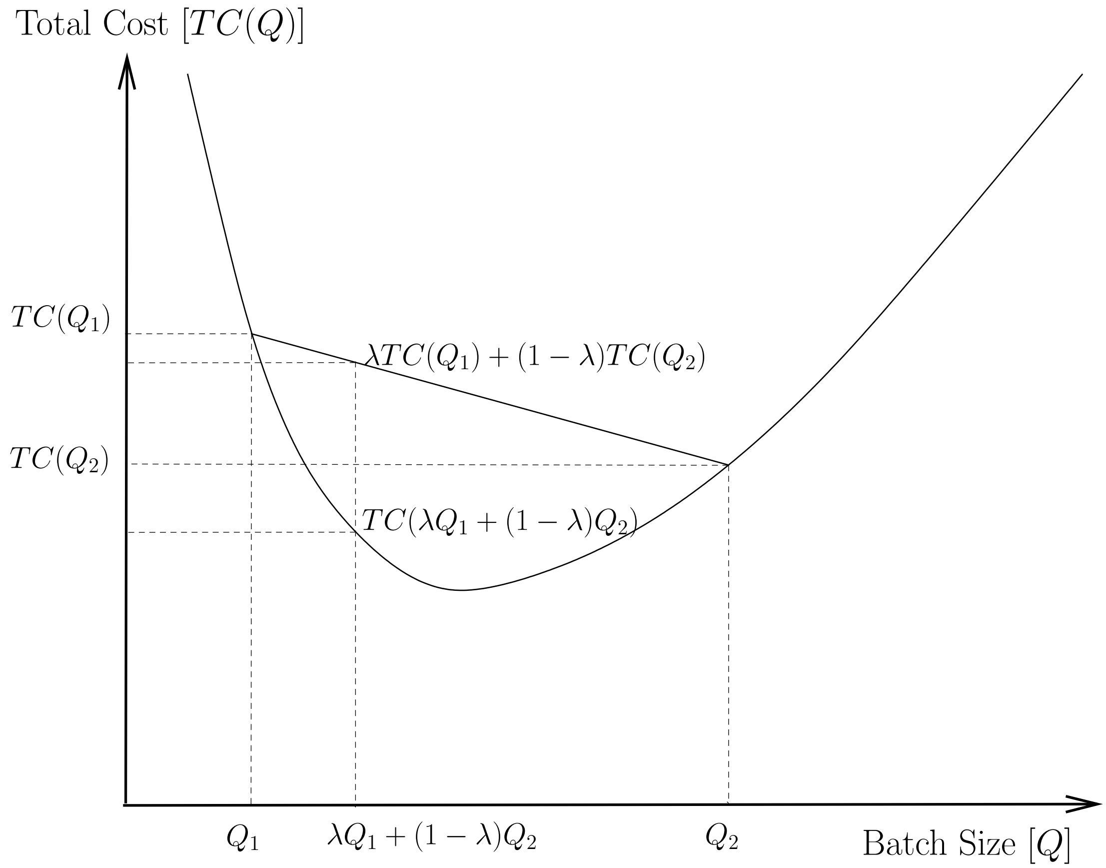

Figure 2: Convexity of the total cost function $TC(Q)$

Though it is clear from Fig. 1 that the total cost function is convex, we can algebraically prove that it is the case. When the total cost function is convex, the optimal order quantity is unique. Consider the graph in Fig 2. For convexity, the following needs to hold:

$$\lambda TC(Q_1) + (1-\lambda)TC(Q_2) \geq TC(\lambda Q_1 + (1-\lambda)Q_2) \tag{11}$$

where $Q_1, Q_2 \geq 0$, $Q_1 \neq Q_2$ and $0 < \lambda < 1$. Writing out the total cost functions fully in Eq. 11, we get:

$$\frac{\lambda SD}{Q_1} + \frac{\lambda h\rho Q_1}{2} + \frac{(1-\lambda)SD}{Q_2} + \frac{(1-\lambda)h\rho Q_2}{2} \geq \frac{SD}{\lambda Q_1 + (1-\lambda)Q_2} + \frac{h\rho[\lambda Q_1 + (1-\lambda)Q_2]}{2} \tag{12}$$

Simplifying further, we obtain:

$$\begin{aligned}
&\frac{\lambda Q_2 + (1-\lambda)Q_1}{Q_1 Q_2} \geq \frac{1}{\lambda Q_1 + (1-\lambda)Q_2} \\
&\lambda^2 Q_1 Q_2 + \lambda(1-\lambda)Q_2^2 + \lambda(1-\lambda)Q_1^2 + (1-\lambda)^2 Q_1 Q_2 \geq Q_1 Q_2 \\
&2\lambda(\lambda-1)Q_1 Q_2 + \lambda(1-\lambda)Q_2^2 + \lambda(1-\lambda)Q_1^2 \geq 0 \\
&(Q_1 - Q_2)^2 \geq 0
\end{aligned} \tag{13}$$

Therefore, $TC(Q)$ is (strictly) convex, i.e. the inequality holds strictly, and Q^* is the unique minimum.

The EPQ Model with Planned Backorders

We will now apply the proposed method to the EPQ model with planned backorders. The following are the additional variables and parameters for the backordering model:

$$\begin{aligned}
b &= \text{the cost of backordering per unit per unit time} \\
B &= \text{the number of units to backorder in each ordering cycle} \\
T_I &= \text{positive inventory time in each cycle (inventory period)} \\
T_B &= \text{zero inventory time in each cycle (backorder period)}
\end{aligned}$$

In the EPQ model with backorders, the inventory cycle starts with an accumulated backorder quantity of B and the backorders are fulfilled at the rate of $P - D$, until they reach zero. Then, the inventory builds up at the same rate until the batch of Q units has been produced, at this time, the inventory will have reached its maximum level. From then on, it depletes at a rate of D until the inventory level reaches zero. From then until the end of the cycle, backorders accumulate until reaching B. Then, the maximum inventory level can be calculated as follows:

$$I_{max} = \frac{Q}{P}(P - D) - B = \rho Q - B \tag{14}$$

The inventory and backorder periods T_I and T_B can be calculated as follows:

$$T_I = \frac{Q}{D} - \frac{B}{D} - \frac{B}{P-D} = \frac{Q}{D} - \frac{B}{\rho D} = \frac{\rho Q - B}{\rho D} \tag{15}$$

$$T_B = \frac{B}{P-D} + \frac{B}{D} = \frac{BP}{D(P-D)} = \frac{B}{\rho D} \tag{16}$$

The average inventory and backorder levels per cycle can be determined as follows:

$$\bar{I} = \frac{\rho Q - B}{2}\left(\frac{T_I}{T}\right) = \frac{\rho Q - B}{2}\left(\frac{\rho Q - B}{\rho Q}\right) = \frac{(\rho Q - B)^2}{2\rho Q} \tag{17}$$

$$\bar{B} = \frac{B}{2}\left(\frac{T_B}{T}\right) = \frac{B}{2}\left(\frac{B}{\rho Q}\right) = \frac{B^2}{2\rho Q} \tag{18}$$

Then, the average unit time total cost will be as follows:

$$TC(Q, B) = \frac{SD}{Q} + \frac{h(\rho Q - B)^2}{2\rho Q} + \frac{bB^2}{2\rho Q} \tag{19}$$

Let B^* be the optimal backorder quantity and let:

$$B_u = B^* + \Delta B \tag{20}$$

$$B_l = B^* - \Delta B \tag{21}$$

for some $\Delta B > 0$. Then, the following will hold:

$$\begin{aligned}
&TC(Q, B_u) - TC(Q, B^*) \geq 0 \\
&\frac{SD}{Q} + \frac{h(\rho Q - B_u)^2}{2\rho Q} + \frac{bB_u^2}{2\rho Q} - \frac{SD}{Q} - \frac{h(\rho Q - B^*)^2}{2\rho Q} - \frac{b(B^*)^2}{2\rho Q} \geq 0 \\
&\frac{SD}{Q} + \frac{h(\rho Q - B_u)^2}{2\rho Q} + \frac{bB_u^2}{2\rho Q} - \frac{SD}{Q} - \frac{h(\rho Q - B^*)^2}{2\rho Q} - \frac{b(B^*)^2}{2\rho Q} \geq 0 \\
&\frac{h}{2\rho Q}\left[(\rho Q - B_u)^2 - (\rho Q - B^*)^2\right] + \frac{b}{2\rho Q}(B_u + B^*)(B_u - B^*) \geq 0 \\
&h(2\rho Q - B_u - B^*)(B^* - B_u) + b(B_u + B^*)(B_u - B^*) \geq 0 \\
&-h(2\rho Q - B_u - B^*) + b(B_u + B^*) \geq 0
\end{aligned} \tag{22}$$

The following will also hold:

$$\begin{aligned}
&TC(Q, B_l) - TC(Q, B^*) \geq 0 \\
&\frac{SD}{Q} + \frac{h(\rho Q - B_l)^2}{2\rho Q} + \frac{bB_l^2}{2\rho Q} - \frac{SD}{Q} - \frac{h(\rho Q - B^*)^2}{2\rho Q} - \frac{b(B^*)^2}{2\rho Q} \geq 0 \\
&\frac{SD}{Q} + \frac{h(\rho Q - B_l)^2}{2\rho Q} + \frac{bB_l^2}{2\rho Q} - \frac{SD}{Q} - \frac{h(\rho Q - B^*)^2}{2\rho Q} - \frac{b(B^*)^2}{2\rho Q} \geq 0 \\
&\frac{h}{2\rho Q}\left[(\rho Q - B_l)^2 - (\rho Q - B^*)^2\right] + \frac{b}{2\rho Q}(B_l + B^*)(B_l - B^*) \geq 0 \\
&h(2\rho Q - B_l - B^*)(B^* - B_l) + b(B_l + B^*)(B_l - B^*) \leq 0 \\
&-h(2\rho Q - B_l - B^*) + b(B_l + B^*) \leq 0
\end{aligned} \tag{23}$$

Thus, Eqs. 22 and 23 result in the following inequality:

$$-h(2\rho Q - B_u - B^*) + b(B_u + B^*) \geq 0 \geq -h(2\rho Q - B_l - B^*) + b(B_l + B^*) \tag{24}$$

As we decrease ΔB, approaching $\Delta B = 0$, both B_l and B_u approach B^*. Furthermore, the two sides of Eq. 24 approach one another. Therefore, when ΔB is approaching zero, the following holds:

$$-h(2\rho Q - 2B^*) + 2bB^* = 0$$

$$B^* = \rho Q\left(\frac{h}{b+h}\right) = Q\left(\frac{P-D}{P}\right)\left(\frac{h}{b+h}\right) \tag{25}$$

We can eliminate the backordering variable B from Eq. 19 by substituting Eq. 25 in Eq. 19:

$$TC(Q) = \frac{SD}{Q} + \frac{h\left(\rho Q - \rho\frac{h}{b+h}Q\right)^2}{2\rho Q} + \frac{b\rho^2 Q^2 \frac{h^2}{(b+h)^2}}{2\rho Q} = \frac{SD}{Q} + \frac{h\rho b^2 Q + b\rho h^2 Q}{2(b+h)^2}$$

$$TC(Q) = \frac{SD}{Q} + \frac{h\rho b(b+h)Q}{2(b+h)^2} = \frac{SD}{Q} + \frac{h\rho bQ}{2(b+h)} \tag{26}$$

Eq. 26 has the same form as Eq. 2. The only difference between the two is the coefficient of the second term. Holding cost rate $h\rho$ in Eq. 2 is replaced by $h\rho\left(\frac{b}{b+h}\right)$. Therefore, the optimal order quantity will be as follows:

$$Q^* = \sqrt{\frac{2SD}{h}}\sqrt{\frac{P}{P-D}}\sqrt{\frac{b+h}{b}} \tag{27}$$

Proving the Uniqueness of the Optimal Solution

In order to prove that (Q^*, B^*) is the unique minimum, we need to show that the following equation holds for $Q_1, Q_2 \geq 0$, $Q_1 \neq Q_2$; $B_1, B_2 \geq 0$, $B_1 \neq B_2$; and $0 < \lambda < 1$:

$$\lambda TC(Q_1, B_1) + (1-\lambda)TC(Q_2, B_2) \geq$$
$$TC(\lambda Q_1 + (1-\lambda)Q_2, \lambda B_1 + (1-\lambda)B_2) \tag{28}$$

First, we will transform Eq. 19 into the following equivalent form:

$$TC(Q, B) = \frac{SD}{Q} + \frac{h(\rho Q - 2B)}{2} + \frac{(h+b)B^2}{2\rho Q} \tag{29}$$

The first term in the above is obviously strictly convex, the second term is linear and therefore both convex and concave. The third term should satisfy the following:

$$\frac{\lambda(h+b)B_1^2}{2\rho Q_1} + \frac{(1-\lambda)(h+b)B_2^2}{2\rho Q_2} \geq \frac{(h+b)(\lambda B_1 + (1-\lambda)B_2)^2}{2\rho(\lambda Q_1 + (1-\lambda)Q_2)} \tag{30}$$

This can be simplified as follows:

$$\frac{\lambda Q_2 B_1^2 + (1-\lambda)Q_1 B_2^2}{Q_1 Q_2} \geq \frac{\lambda^2 B_1^2 + 2\lambda(1-\lambda)B_1 B_2 + (1-\lambda)^2 B_2^2}{\lambda Q_1 + (1-\lambda)Q_2}$$
$$\lambda^2 Q_1 Q_2 B_1^2 + \lambda(1-\lambda)Q_2^2 B_1^2 + \lambda(1-\lambda)Q_1^2 B_2^2 + (1-\lambda)^2 Q_1 Q_2 B_2^2 \geq$$
$$\lambda^2 Q_1 Q_2 B_1^2 + 2\lambda(1-\lambda)Q_1 Q_2 B_1 B_2 + (1-\lambda)^2 Q_1 Q_2 B_2^2$$
$$Q_2^2 B_1^2 + Q_1^2 B_2^2 - 2Q_1 Q_2 B_1 B_2 \geq 0 \Rightarrow (Q_2 B_1 - Q_1 B_2)^2 > 0$$

Thus, the third term is also strictly convex. Therefore, $TC(Q, B)$ is strictly convex and (Q^*, B^*) is the unique optimum solution to the EPQ problem with backordering.

CONCLUSIONS

In this paper, we demonstrate a simple method to derive the optimal solution for the EPQ model with backordering, without using any calculus terms or concepts. The method is entirely based on simple algebra and analytic geometry, and it is suitable to teach the model to students who lack sufficient preparation in calculus. We also demonstrate that the optimal solution is unique, without using calculus. Contrary to the existing algebraic methods in the literature, our method is much simpler and much more succinct, and does not require a priori knowledge of the characteristics of the optimal solution. It has great potential as a pedagogical tool to teach inventory management to students who are not well-versed in differential calculus.

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