PREVENTIVE REPLACEMENT WITH RANDOM PRODUCTION RATE

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ABSTRACT

A mathematical model is presented to find the optimal timing of preventive replacement of equipment with random production rate. If the equipment is replaced too early, the future production volume is lost, and if too late, because of the operation cost and decreasing production rate the operation of the equipment becomes economically ineffective. The life cycle cost and benefit per unit time are random, the net cost has to be minimized. Both its expectation and variance are minimized leading to a multi objective programming problem, which is solved to find the optimal balance between cost and benefit during an operation cycle.

Keywords: Preventive Replacement, Reliability Analysis, Multi-objective Optimization Problem

1. INTRODUCTION

The main tasks of reliability engineering include inspection scheduling, maintenance, repairs and even replacement. In the case of many equipment repairs are not possible, only failure replacement has to be performed in cases of failures. Their costs are usually much higher than those of preventive replacement, when the management does not wait until failure occurs in order to avoid higher replacement costs and economically ineffective operation. Therefore, finding the optimal replacement timing, a balance has to be determined between operation and replacement costs, and economic benefit.
The problem is stochastic, since the times when failures occur are not known, only their probabilistic characterization is possible based on past failures of identical equipment.
The fundamentals of reliability engineering are discussed in many textbooks and monographs including Barlow and Proschan (1965), and Nakagawa and Zhao (2015). Similarly, the used probability methodology can be found for example in Milton and Arnolds (2003) and Ross (2000).

2. THE MATHEMATICAL MODEL

This paper considers an equipment which is subject to random failure, which cannot be repaired. If it occurs, then the equipment has to be replaced with a new one. Since its productivity declines in time, the equipment has to be replaced before it becomes non profitable. Therefore, the management selects a time when the equipment will be replaced if it is still in working condition. If it breaks down before the scheduled replacement, then providing unscheduled manpower, tools and considering additional damages the total cost and damages might become high. Therefore, the management wants to find the scheduled time when preventive replacement has to be performed even if the equipment is still in working condition.

The time x of possible failure is a random variable with cumulative distribution function $F(x)$ and pdf $f(x)$. Here $F(x)$ gives the probability that failure will occur before time x. Let $R(x) = 1 - F(x)$ denote the reliability function giving the probability that at the time x the equipment still works. The equipment produces revenue with

$$Q(t) = A - Bt^2 \tag{1}$$

output rate showing that it declines in time. The cost of preventive replacement is denoted by c_1 and that of failure replacement is $c_2 > c_1$. In addition, let t be the time of the scheduled replacement and α the unit operation cost of the equipment. The net cost per unit time in a cycle is also random:

$$g(t,x) = \begin{cases} g_1(t) = \frac{\alpha t + c_1 - \int_0^t Q(x)dx}{t}, & if\ x \geq t. \\ g_2(x) = \frac{\alpha x + c_2 - \int_0^x Q(x)dx}{x}, & if\ x < t. \end{cases} \tag{2}$$

Using the actual form of $Q(x)$ we have

$$g_1(t) = \frac{1}{t}(\ \alpha t + c_1 - At + B\frac{t^3}{3}) \tag{3}$$

and

$$g_2(x) = \frac{1}{x}(\ \alpha x + c_2 - Ax + B\frac{x^3}{3}) \tag{4}$$

Since the time of failure x is random, we have two possibilities: $X > t$ or $X < t$. Therefore, the expectation of $g(t,x)$ with respect to x has two terms:

$$\bar{g}(t) = \frac{1}{t}\left(\ \alpha t + c_1 - At + B\frac{t^3}{3}\right)R(t) + \int_0^t \frac{1}{x}(\ \alpha x + c_2 - Ax + B\frac{x^3}{3})f(x)dx \tag{5}$$

This is the function what the management has to minimize. Notice first that this objective function can be rewritten as

$$\bar{g}(t) = \alpha - A + \frac{c_1}{t} + \int_0^t(\ \alpha - A + \frac{c_2}{x})\ f(x)dx + B(\frac{t^2}{3}R(t) + \int_0^t \frac{x^2}{3}\ f(x)dx\ ) \tag{6}$$

Assume next that the management is not certain in the value of the decline coefficient B. So $\bar{g}(t)$ is also random. Let $\bar{B}$ denote the expectation of B, and σ^2 its variance. Then clearly

$$E[\bar{g}(t)] = \alpha - A + \frac{c_1}{t} + \int_0^t(\ \alpha - A + \frac{c_2}{x})\ f(x)dx + \bar{B}(\frac{t^2}{3}R(t) + \int_0^t \frac{x^2}{3}\ f(x)dx) \tag{7}$$

and

$$Var\left(\bar{g}(t)\right) = \ \sigma^2(\frac{t^2}{3}R(t) + \int_0^t \frac{x^2}{3}\ f(x)dx\ )^2 \tag{8}$$

The management has now two objectives. At one hand it is willing to minimize the expected net cost and at the same time they want to minimize the risk by minimizing the variance of the net cost. So, a multi-objective optimization problem is obtained with objective functions (7) and (8). Assume that minimizing expectation is K-times less important than minimizing variance, then the management minimizes the composite objective

$$\text{Minimize} \quad E(\,\bar{g}(t)\,) + K\,Var\big(\,\bar{g}(t)\big) \tag{9}$$

The optimal solution of this problem provides the optimal value of the time of the scheduled preventive replacement.

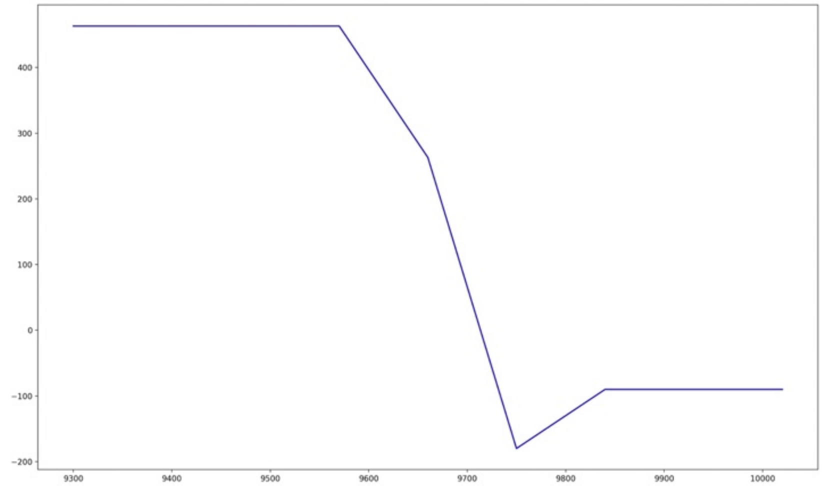
3. NUMERICAL EXAMPLE

Consider an equipment with a Weibull failure rate, $F(x) = 1 - e^{-(\frac{x}{2})^2}$. In addition, assume that $\alpha = 10$, $A = 100, \bar{B} = 150$, $\sigma = 10$ and the replacement costs are $c_1 = 100$, $c_2 = 500$.
Table 1 shows the values of $E[\bar{g}(t)]$, $Var\big(\bar{g}(t)\big)$ for specific time. The composite objective function (9) is tabulated in the last row of each segment by assuming $K = 0.5$.
To solve Equation (9), a time interval is to be considered for finding optimal *t*. Therefore, in this problem (1,10,000) time interval is studied to find the optimal time to have a preventive replacement. As it is shown in Figure 1 and Table 1 the minimum value of Equation (9) took placed in t=9787. Therefore, this time is an optimal time to reschedule for a replacement.

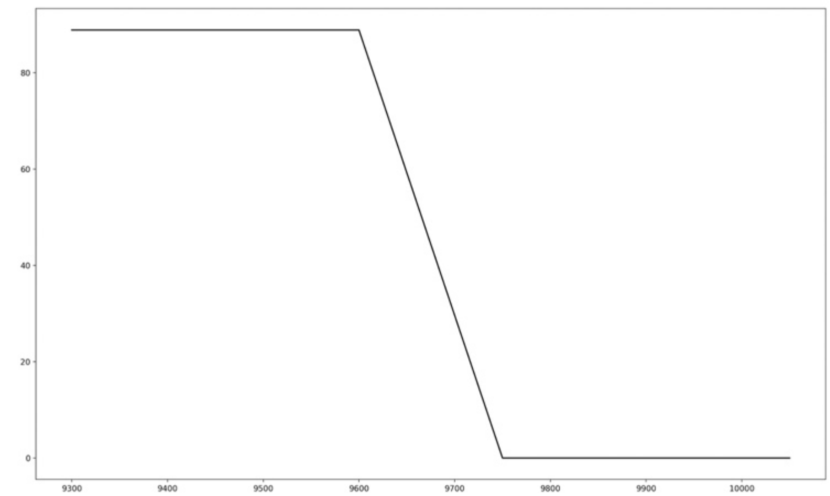
Table 1. The expected $\bar{g}(t)$,variance, composite function values in specific time interval of (1,10,000).

Time	1	1000	2000	3000	4000	5000
Equation (7)	264.972	463.213	463.163	463.146	463.138	463.1334
Equation (8)	8.698	177.777	177.777	177.777	177.777	177.777
Equation (9)	269.321	552.102	552.052	552.035	552.027	552.022
Time	6000	7000	8000	9000	9787	10000
Equation (7)	463.130	463.127	463.125	463.124	-179.989	-89.989
Equation (8)	177.777	177.777	177.777	177.777	9.831e-16	9.846e-17
Equation (9)	552.019	552.016	552.014	552.013	-179.989	-89.989

(a)



(b)



(c)

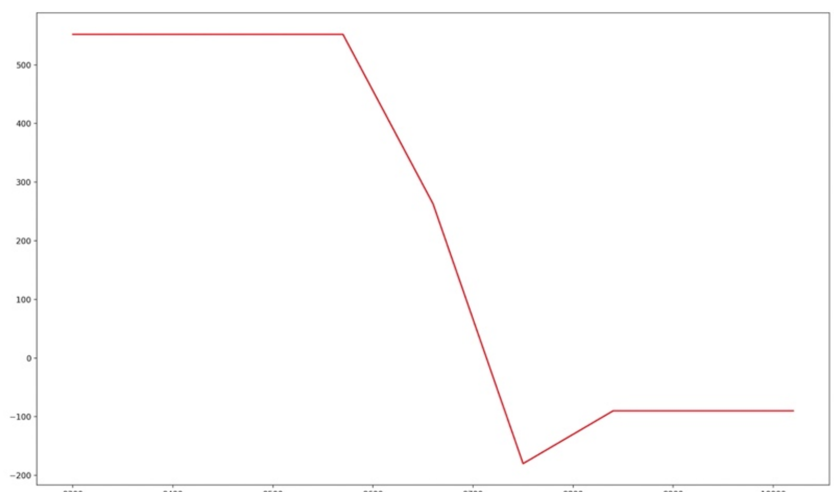


Fig 1. The expected $\bar{g}(t)$,variance, composite function a, b and c, respectively.

4. CONCLUSIONS

In this paper the optimal timing of previous replacement of an equipment is determined minimizing the net cost including operating and replacement costs and production benefit. The output rate assumed to be random. The expectation of the net cost as well as its variance are minimized to decrease risk. This multi objective optimization problem is solved by using the weighting method based on subjective preference of the management

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