Using Non-oriented Equilibrium Efficient Frontier DEA Approach to Evaluate Fixed-sum-outputs DMUs

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ABSTRACT

Equilibrium efficient frontier DEA approaches are developed to measure efficiency levels of decision-making units (DMUs) with fixed-sum outputs. However, present approaches in literature only evaluate either input-oriented or output-oriented efficiency. In this paper, we propose a new non-oriented equilibrium efficient frontier DEA (NEEFDEA) approach, it considers input excesses and output shortfalls jointly to assess the non-oriented efficiency of DMUs. Thus, NEEFDEA can obtain more overall and discriminative efficiency results to assist actual managerial decisions. We further employ a Nash bargaining game to optimize the evaluation results. The bargaining-based selection can generate a unique and Pareto-optimal NEEF via one step. Finally, we illustrate this NEEFDEA approach by a numerical example, we also conduct an empirical study of this approach by using data of 30 companies in the vehicle industry in 2020 and compare the performance with some traditional equilibrium frontier DEA approaches.

Keywords: Equilibrium efficient frontier; non-oriented DEA measure; fixed-sum outputs; Nash Bargaining game.

1. Introduction and Literature Review

Data envelopment analysis (DEA) is a nonparametric tool for determining the relative efficiency of homogenous DMUs with multiple inputs and multiple outputs. Dyson et al. [9] and Lins et al. [16] pointed out a pitfall in applying DEA, where outputs are exogenous and constrained. For example, when evaluating the performance of countries in the Olympic Games, the total number of gold, silver, and bronze medals is fixed. If one participating country wins more medals, others can only share the remaining medals. The same can be said for some outputs expressed in proportions, such as market share. Traditional DEA models do not take into account the dependence among inputs and outputs, thus failing to evaluate DMUs with those kinds of outputs, whose sum is fixed.

To deal with the fixed-sum outputs, some novel DEA approaches have been developed in recent years. The first branch of approaches include zero-sum gains DEA models presented by Bandeira et al. [3], Gomes and Lins [11], and Lins et al. [16], as well as the fixed-sum output DEA approach presented by Yang et al. [25]. They use a succession of "no memory" operations to evaluate DMUs

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one by one. That is, the outcome of the previous DMU is ignored while assessing the next one. As a result, DMUs are evaluated based on different frontiers, leading to efficiency results are not strictly comparable. To address the problem, the second branch of approaches, namely equilibrium efficient frontier DEA approaches, has been developed. The idea of these approaches is to firstly search for an equilibrium efficient frontier (EEF) and then treat it as the common benchmark to measure the final efficiency of DMUs. Therefore, there is a major challenge is that how to construct an EEF. Yang et al. [27] initially proposed an equilibrium efficient frontier DEA (EEFDEA) approach, which adjusts the fixed-sum outputs of inefficient DMUs in a predetermined turn until all DMUs locates on a common EEF. However, the process of determining an EEF suffers from cumbersome steps and heavy calculation burdens. Following that, Yang et a. [26] proposed the GEEFDEA approach, which can reach an equilibrium efficient state in just one step. In addition, GEEFDEA offers another two benefits over EEFDEA. Firstly, the obtained EEF is independent of the processing sequence of DMUs. Secondly, the sign of adjustments of fixed-sum outputs does not have to be the same for each DMU. These more flexible constraints help to get a more optimal EEF under the minimal adjustment strategy. Fang [10] also improved the EEF achieving phase of EEFDEA and come up with the same model as GEEFDEA. Since GEEFDEA pioneered a convenient way to construct an EEF, many researchers conducted follow-up studies based on it. For example, Amirteimoori et al. [1] constructed multiple context-based EEFs. Wu et al. [24] extend the GEEFDEA to deal with undesirable fixed-sum outputs. Zhu et al. [29] considered fixed-sum inputs and fixed-sum undesirable outputs simultaneously, and constructed a new common EEF by minimizing each pair of DMUs' adjustments of fixed-sum inputs and outputs. Li et al. [13] proposed an equilibrium frontier approach to evaluate performances of two-stage networks with fixed-sum outputs. Li et al. [14] defined the Malmquist productivity index based on an equilibrium frontier approach with undesirable fixed-sum outputs. So far, these equilibrium efficient frontier methods have been applied to analyze the performance of banks (e.g., Amirteimoori et al. [1]), participating nations in Olympics (e.g., Li et al. [13], Yang et al. [26]), industrial sectors (e.g., Wu et al. [23], Zhu et al. [28], Zhu et al. [29]), and appliance companies (e.g., Chen et al. [7], Yang et al. [27]), etc.

In addition to how to construct an EEF, another major challenge of equilibrium efficient frontier DEA approaches is how to deal with the non-unique EEFs [26]. The first branch of studies attempts to select a unique EEF by adding proper secondary goals. For example, Fang [10] introduced the goal of minimizing the maximum relative deviation of each fixed-sum output as well as AR-I type restrictions into EEF achieving models. Zhu et al. [30] treated maximizing all DMUs' efficiency satisfaction degree as secondary goals, getting a multi-objective non-linear EEF achieving model. Unfortunately, these secondary goals usually can only narrow the scope of EEFs, and hence Zhu et al. [31] developed an iterative algorithm to help to achieve uniqueness indeed. Note that these methods cannot ensure the selected EEF is Pareto optimal, thus making it difficult to convince all DMUs to agree with it. On the contrary, Chen et al. [7] try to measure DMUs' efficiency referring to all feasible EEFs, but found that the possible efficiency of each DMU is continuous within an interval, not able to be enumerated. Thus, Chen et al. [7] developed several models to calculate the efficiency intervals, rank intervals, and dominance relations of DMUs. The approach can provide more informative results, but it always fails to rank all DMUs. Besides, it can only be applied when the assumption of the constant returns to scale (CRS) is satisfied.

Although equilibrium efficient frontier approaches have made adequate progress, they still suffer from two major flaws. On the one hand, the existing approaches, as far as we know, can only measure the input-oriented or output-oriented efficiency of DMUs. However, considering the waste of inputs and the shortage of outputs simultaneously can get a more comprehensive evaluation result, offering more useful information to support actual managerial decisions. Furthermore, by combining input-oriented and output-oriented features, non-oriented measurement can get a more discriminating result [21]. Therefore, it is necessary to put forward a non-oriented equilibrium frontier approach from both a practical and theoretical standpoint. On the other hand, these existing methods for handling multiple equilibrium efficient frontier are not ideal. Actually, ranking DMUs by their efficiency is an initial motivation of many DEA-based studies, such as Aparicio and Zofío [2], Liang et al. [15], Kao and Liu [12], and Liu et al. [17], due to the importance of ranking in actual managerial decisions [27]. Therefore, it is necessary to select a rational EEF to calculate exact efficiency values to rank DMUs with fixed-sum outputs. However, there are no clear criteria to guide the selection process of multiple EEFs. Based on different EEFs, the final efficiency values even rankings of DMUs may vary. Thus, finding a Pareto-optimal EEF, balancing the diverse preference on EEF selection among DMUs, is an effective way to solve the problem. Based on such a frontier, a more rational and fair result can be obtained, and DMUs are also able to reach a consensus on it.

Based on the above observations, first, this paper contributes by proposing the NEEFDEA approach, which can measure the non-oriented efficiency of DMUs with fixed-sum outputs to provide more comprehensive evaluation results to assist actual managerial decisions. NEEFDEA is also a two-step approach, the first step of which constructs a non-oriented equilibrium efficient frontier (NEEF), and then the second step assesses the non-oriented efficiency of DMUs based on the NEEF. However, NEEFDEA always generates either one or an infinite number of NEEFs. Therefore, the second contribution of this paper is to present a bargaining-based selection to address the problem of non-unique NEEFs. By incorporating the bargaining game theory, we define the selection process of NEEFs as a bargaining game among DMUs. And then build the bargaining-based selection model based on the famous Nash bargaining solution. The unique NEEF obtained by this model also satisfies Pareto-optimality, which cannot be achieved by prior methods. Such a NEEF can make DMUs reach a consensus on final evaluation results. Finally, this paper uses a numerical example and an empirical case of 30 companies in the vehicle industry in 2020 respectively to illustrate the proposed approaches, and compare them with some traditional equilibrium efficient frontier approaches to reflect the improvements.

The rest of the paper is arranged as follows. In Section 2, we propose the NEEFDEA approach, including NEEF achieving models and NEEF-based evaluation models. Section 3 deals with the problem of non-unique NEEFs. The bargaining-based selection proposed in section 3.2 can pick up a unique and Pareto-optimal NEEF. We use a numerical example to illustrate our approaches, and compare them with traditional methods in Section 4. Section 5 applies our approaches to do an empirical study on 30 companies in the vehicles industry on the Global Fortune 500 list in 2020. Finally, we summarize the conclusions and directions for further research in Section 6.

2. The NEEFDEA Approach

Assume that there are total n DMUs in the system, and each DMU_j $(j = 1,2, \dots, n)$ converts its input vector $\boldsymbol{x_j} = (x_{1j}, \dots, x_{ij}, \dots, x_{mj}) \in \mathbb{R}_+^m$ into a variant-sum output vector $\boldsymbol{y_j} = (y_{1j}, \dots, y_{rj}, \dots, y_{sj}) \in \mathbb{R}_+^s$ and a fixed-sum output vector $\boldsymbol{f_j} = (f_{1j}, \dots, f_{tj}, \dots, f_{lj}) \in \mathbb{R}_+^l$. Besides, the fixed-sum outputs satisfy that $\sum_{j=1}^{n} f_{tj} = F_t$ for $\forall\, t$, where F_t is a constant. The characteristic of fixed-sum outputs determines that DMUs will affect each other. The conventional directional distance function (DDF) is presented by Chambers et al. [4] to evaluate the non-oriented efficiency levels of DMUs, shown as bellows:

$$
\begin{aligned}
&\text{Max } \theta_k \\
&s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ik} - \theta_k x_{rk}, \forall i \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rk} + \theta_k y_{rk}, \forall r \\
&\sum_{j=1}^{n} \lambda_j f_{tj} \ge f_{tk} + \theta_k f_{tk}, \forall t \\
&\sum_{j=1}^{n} \lambda_j = 1, \\
&\lambda_j \ge 0, \theta_k \text{ is free},
\end{aligned}
\tag{1}
$$

where $(\lambda_1, \dots, \lambda_j, \dots, \lambda_n)$ describes the composing structure of the reference point on the efficient frontier of DMU_k. The model makes the object DMU_k become at least as efficient as others by reducing its inputs and increasing its outputs simultaneously and finally join the efficient frontier. Therefore, θ_k reflects the potential of DMU_k for efficiency improvement, that is, the non-oriented inefficiency of DMU_k. Evidently, when $\theta_k = 0$, DMU_k is overall efficient. However, DDF does not take into account the dependence among DMUs, thus failing to evaluate the efficiency levels of DMUs in this situation [16] [27].

To evaluate the non-oriented efficiency levels of DMUs with fixed-sum outputs, we propose the NEEFDEA approach in this Section. Following the idea of traditional equilibrium efficient frontier DEA approaches, NEEFDEA also has two steps. The first step aims to find an equilibrium efficient state, where all DMUs synchronously become efficient by adjusting fixed-sum outputs. In this paper, we require that all DMUs are efficient both in inputs and in outputs, and the frontier on which they are located is defined as the non-oriented equilibrium efficient frontier (NEEF). Then, by treating the NEEF as a common benchmark, the second step evaluates the non-oriented efficiency levels of DMUs. Correspondingly, NEEFDEA includes two kinds of models, i.e., NEEF achieving models in **Section 2.1** and NEEF-based evaluation models in **Section 2.2**. As the name implies, the role of the former models is to construct a NEEF in the first step, and the latter models are used in the second step to assess the non-oriented efficiency of DMUs.

2.1 NEEF achieving models

When determining a non-oriented equilibrium efficient state, a crucial problem is how to represent the non-oriented efficiency of all DMUs simultaneously. Although θ_k can measure the non-oriented inefficiency of each DMU, it is quite difficult to use it to depict the non-oriented equilibrium efficient state. Because the feasible region of model (1) changes when dealing with different evaluated DMUs. To address the problem, we convert model (1) to an equivalent model by using dual transformation and Charnes-Cooper transformation [5], shown as below:

$$\text{Min}\ \frac{\sum_{i=1}^{m} v_i x_{ik} - \sum_{r=1}^{s} u_r y_{rk} - \sum_{t=1}^{l} w_t f_{tk} + \mu_0}{\sum_{i=1}^{m} v_i x_{ik} + \sum_{r=1}^{s} u_r y_{rk} + \sum_{t=1}^{l} w_t f_{tk}}$$
$$s.t.\ \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} - \sum_{t=1}^{l} w_t f_{tj} + \mu_0 \geq 0, \forall j \qquad (2)$$
$$v_i, u_r, w_t \geq 0, \mu_0\ is\ free,$$

where v_i, u_r, w_t are non-negative weights of the ith input, the rth variant-sum output, and the tth fixed-sum output respectively. The objective function is an equivalent conversion of θ_k, still denoting the non-oriented inefficiency of DMU_k. Adopting this fractional expression, all DMUs' non-oriented inefficiency can be measured within the same feasible zone.

Based on model (2), we build the NEEF achieving model to construct a NEEF as bellows:

$$Min \sum_{j=1}^{n} \sum_{t=1}^{l} w_t \left|\delta_{tj}\right|$$
$$s.t.\ \frac{\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} - \sum_{t=1}^{l} w_t \left(f_{tj} + \delta_{tj}\right) + \mu_0}{\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + \sum_{t=1}^{l} w_t \left(f_{tj} + \delta_{tj}\right)} = 0, \forall j$$
$$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} - \sum_{t=1}^{l} w_t \left(f_{tj} + \delta_{tj}\right) + \mu_0 \geq 0, \forall j \qquad (3)$$
$$\sum_{j=1}^{n} \delta_{tj} = 0, \forall t$$
$$f_{tj} + \delta_{tj} \geq 0, \forall t, \forall j$$
$$v_i, u_r, w_t \geq 0, \forall i, \forall r, \forall t$$
$$\delta_{tj}, is\ free, \forall t, \forall j$$
$$\mu_0\ is\ free$$

where δ_{tj} is the adjustment of the tth fixed-sum output of DMU_j. When $\delta_{tj} < 0\ (> 0)$, DMU_j has to reduce (increase) its tth fixed-sum output to reach the equilibrium efficient state. $\delta_{tj} = 0$ means the tth fixed-sum output of DMU_j can remain unchanged. The first two sets of constraints are used to portray the equilibrium state. More specifically, the first set makes all DMUs to become overall efficient after the adjustment. The second set ensures the input-output structures of adjusted DMUs are rational, but these constraints are actually redundant and can be removed. The third set of constraints makes sure that the adjustment does not change the sum of each fixed-sum output. The fourth set of constraints guarantees all adjusted fixed-sum outputs are non-negative. Therefore, these two sets of constraints jointly ensure the rationality of the adjustment scheme. As for the objective function, it follows the minimal adjustment strategy proposed by Yang et al. [26]. In this way, DMUs can find the easiest path to reach an equilibrium state through free competition among peers. In addition, model (3) satisfies the VRS assumption. When setting $\mu_0 = 0$, model (3) satisfies the CRS assumption.

Assume $\left(v_i^*, u_r^*, w_t^*, \delta_{tj}^*, \mu_0^*\right)$ is an optimal solution of model (3), which determines a NEEF. More specifically, the fixed-sum outputs of DMUs adjust to $f_{tj} + \delta_{tj}^*(\forall t \forall j)$, and the inputs and rest outputs keep unchanged. These virtual DMUs are efficient both in inputs and in outputs at the same time. Thus, they form a NEEF, and such frontier satisfies the condition that each fixed-sum output maintains a constant sum. As seen from **Theorem 1**, model (3) can always generate a NEEF. Furthermore, **Theorem 2** demonstrates that the adjusted direction of all fixed-sum outputs for each DMU must be the same. That means when existing one $\delta_{tj}^* > 0 (< 0)$, DMU_j lies inside (outside) the NEEF. If all $\delta_{tj}^* (\forall t)$ are equal to zero, DMU_j is exactly located on the NEEF.

Theorem 1. Model (3) is always feasible.

Proof. see Appendix A.

Theorem 2. Assume $(v_i^*, u_r^*, w_t^*, \delta_{tj}^*, \mu_0^*)$ is an optimal solution of model (3), then the adjustments of all fixed-sum outputs of DMU_j always satisfy that $\delta_{tj}^* \geq 0 (\forall t)$ or $\delta_{tj}^* \leq 0 (\forall t)$.

Proof. see Appendix B.

However, model (3) is a non-linear programming model, making it difficult to solve. To lessen the computational burden, we set $\delta'_{tj} = w_t \delta_{tj}, a_{tj} = \frac{1}{2}(|\delta'_{tj}| + \delta'_{tj}), b_{tj} = \frac{1}{2}(|\delta'_{tj}| - \delta'_{tj})$ following Si et al. [22]. Then the original non-oriented equilibrium achieving model can be converted into a linear model as follows:

$$
\begin{aligned}
& Min \sum_{j=1}^{n} \sum_{t=1}^{l} (a_{tj} + b_{tj}) \\
& s.t. \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} - \sum_{t=1}^{l} (w_t f_{tj} + a_{tj} - b_{tj}) + \mu_0 = 0, \forall j \\
& \sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + \sum_{t=1}^{l} (w_t f_{tj} + a_{tj} - b_{tj}) \geq 1, \forall j \\
& \sum\nolimits_{j=1}^{n} a_{tj} = \sum\nolimits_{j=1}^{n} b_{tj}, \forall t \\
& w_t f_{tj} + a_{tj} - b_{tj} \geq 0, \forall t, j \\
& v_i, u_r, w_t, a_{tj}, b_{tj} \geq 0, \mu_0 \text{ is free}
\end{aligned}
\tag{4}
$$

where the second set of constraints is added to rule out the trivial solution. Because $\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + \sum_{t=1}^{l} (w_t f_{tj} + a_{tj} - b_{tj})$ is a denominator in the model (3), implying that it cannot equal to zero. Let $(v_i^*, u_r^*, w_t^*, a_{tj}^*, b_{tj}^*, \mu_0^*)$ denotes the optimal solution of model (4). Then the tth fixed-sum output of DMU_j is adjusted to $\frac{(a_{tj}^* - b_{tj}^*)}{w_t^*}, \forall t, \forall j$. As shown in **Theorem 3**, the addition of the second set of constraints does not affect the optimal adjustment scheme, i.e., $\delta_{tj}^* = \frac{(a_{tj}^* - b_{tj}^*)}{w_t^*}, \forall t, \forall j$.

Theorem 3. The optimal adjustment scheme of fixed-sum outputs obtained from model (4) is identical to that from model (3).

Proof. see Appendix C.

Note that there are total $(m+s+l+2nl+1)$ variables and $(2n+l+nl)$ constraints in model (4). When $(m+s+nl+1)>2n$, model (4) has flexibility in determining an optimal solution. In other words, the NEEF achieving models may also face the challenge of non-unique NEEFs.

2.2 NEEF-based evaluation models

Next comes the second step of NEEFDE, each DMU with its original fixed-sum outputs can be appraised via the NEEF-based evaluation model as follows:

$$
\begin{aligned}
& e_k^{NEEFDEA} = \text{Max}\, \beta_k \\
& s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le (1-\ \beta_k) x_{ik}, \forall i \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge (1+\ \beta_k) y_{rk}, \forall r \\
& \sum_{j=1}^{n} \lambda_j \left(f_{tj} + \delta_{tj}^{*}\right) \ge (1+\ \beta_k) f_{tk}, \forall t \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \ge 0, \beta_k\ is\ free.
\end{aligned}
\tag{5}
$$

where optimally adjusted DMUs constitute the reference set in the left of all inequality constraints. Thus, DMU_k with original inputs and outputs is projected to the NEEF through shrinking its inputs and expanding its outputs in the same proportion. $e_k^{NEEFDEA}$ is the non-oriented inefficiency of DMU_k, taking values in $(-1,1)$. Evidently, when $e_k^{NEEFDEA}=0$, DMU_k has the same efficiency level as the NEEF. $0<e_k^{NEEFDEA}<1$ means that the efficiency level of DMU_k is inferior to the NEEF. For example, if $e_k^{NEEFDEA}=0.2$, DMU_k has to reduce its inputs by 20% and expand its outputs by 20% at the same time to achieve the equilibrium efficiency. While $-1<e_k^{NEEFDEA}<0$, the situation is opposite, and DMU_k performs better than the equilibrium efficiency. Consequently, we can rank all DMUs in terms of $e_k^{NEEFDEA}$.

Theorem 4. Model (5) is always feasible.

Proof. see Appendix D.

For convenience, we set $\tilde{\beta}_k = 1-\ \beta_k$ to get a non-oriented efficiency index $\tilde{e}_k^{NEEFDEA}$, which can be calculated by model (6). Obviously, the value range of $\tilde{e}_k^{NEEFDEA}$ is $(0,2)$, and a larger value means a higher efficiency level.

$$\tilde{e}_k^{NEEFDEA} = \text{Min } \tilde{\beta}_k$$
$$s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le \tilde{\beta}_k x_{ik}, \forall i$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge (2 - \tilde{\beta}_k) y_{rk}, \forall r$$
$$\sum_{j=1}^{n} \lambda_j (f_{tj} + \delta_{tj}^*) \ge (2 - \tilde{\beta}_k) f_{tk}, \forall t \qquad (6)$$
$$\sum_{j=1}^{n} \lambda_j = 1,$$
$$\lambda_j \ge 0, \tilde{\beta}_k \text{ is free.}$$

So far, the NEEFDEA approach is completed. While previous equilibrium efficient frontier approaches only evaluate input-oriented or output-oriented efficiency levels of DMUs, NEEFDEA is non-oriented in both steps, thus able to rank DMUs by capturing their inefficiency in inputs and in outputs simultaneously. In addition, in the step of constructing a NEEF, NEEFDEA can achieve an equilibrium efficient state via one step, inheriting the advantages of GEEFDEA. Meanwhile, NEEFDEA also faces the challenge of non-unique NEEFs, i.e., having flexibility in determining δ_{tj}^* as mentioned above. The traditional secondary goal methods can be used to select a unique NEEF. However, the chosen NEEF does not satisfy Pareto efficiency, resulting in that DMUs disagreeing on the final evaluation results. Thus, we proposed a new approach to select a unique and Pareto-optimal NEEF in Section 4.

3. Empirical Application to 30 Companies in the Vehicle Industry

This section applies the proposed approaches to evaluate the performance of 30 companies in the vehicle industry in 2020. These companies are all on the Global Fortune 500 list, located in 8 countries, that is, JPN, CHN, DEU, KOR, FRA, USA, CAN, and SWE. Each company has two inputs, i.e., total assets (X1) and employees (X2), and two outputs i.e., market share (FY1) and profits (FY2). In this paper, we regard both outputs as fixed-sum outputs. It is because the vehicle industry has a constant market share and profit in one year. The data comes from China Fortune 500 (https://www.fortunechina.com), which is detailed in Appendix H. For convenience, we number these companies as C1-C30.

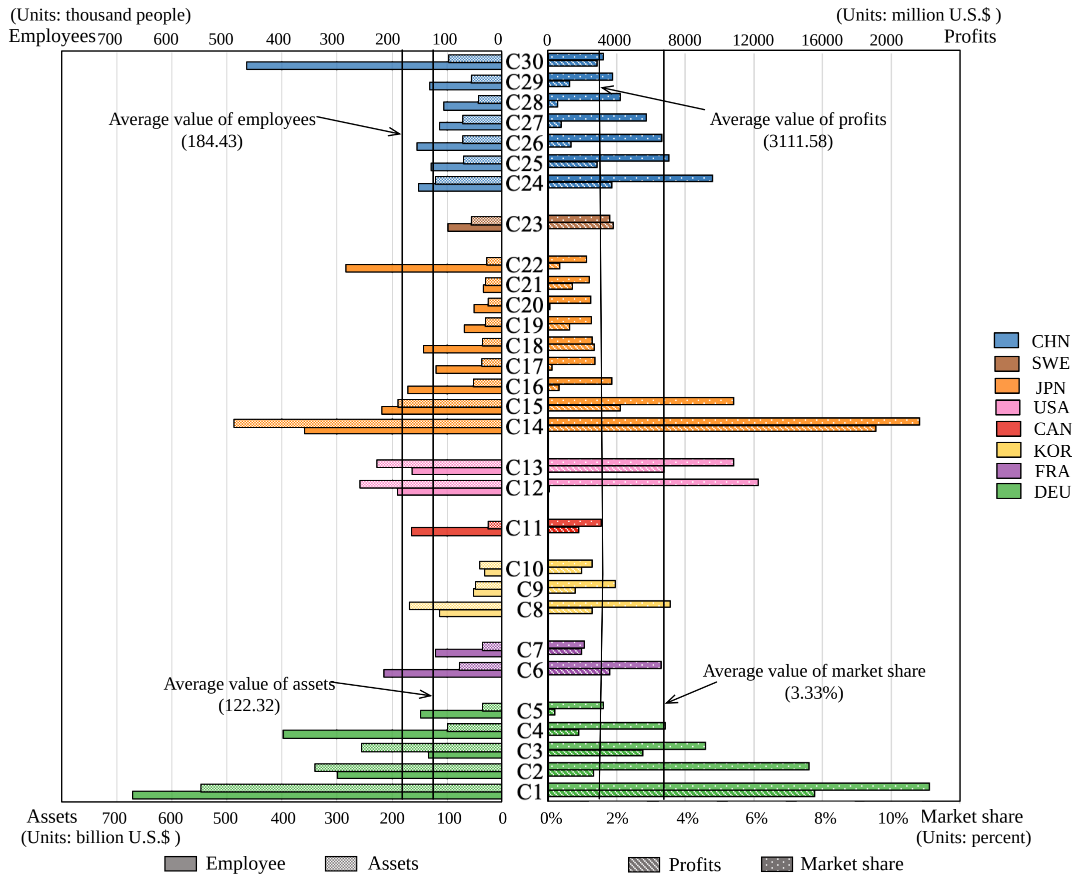

Figure 2. Inputs and outputs for 30 companies in the vehicle industry in 2020.

It can be observed from **Figure 2** that C1 has the largest sizes of both inputs among all companies, followed by C14. Meanwhile, outputs of the both companies are also much more than that of other companies. On the contrary, the inputs of some firms in JPN are less, such as C19, C20, and C21. As for fixed-sum outputs, C12 has the lowest profits and C7 occupies the smallest market share. Considering that the inputs and outputs of 30 companies vary greatly, assume the companies satisfy the VRS assumption to assess their performance. Columns 3 to 6 of **Table 5** show the adjustment ranges of fixed-sum outputs of all companies. Among these companies, C1 and C2 have no flexibility in adjusting fixed-sum outputs, and thus do not take part in the following bargaining problem of selecting a unique and Pareto-optimal NEEF. The bargaining results of Model (12) are shown in columns 7 to 8 of **Table 5**. In addition to C1 and C2, C12 also does not need to adjust its fixed-sum outputs, and hence the three companies are right located on the selected NEEF. As for other companies, 15 of them need to drive some fixed-sum outputs to the rest 12 companies, eventually joining the equilibrium state together. The selected NEEF can be regarded as an ideal state of the whole industry, where all companies have efficient operations and balanced development. Unfortunately, most companies are inferior to the ideal state.

Table 5 The results of the NEEFDEA with bargaining-based selection

Nation	Company	Adjustment range of FY1		Adjustment range of FY2		Bargaining-based adjustment		Non-oriented efficiency	Rank

		UB	LB	UB	LB	δ^* of FY1	δ^* of FY2		
DEU	C1	0	0	0	0	0.00	0.00%	1.000	13
DEU	C2	0	0	0	0	0.00	0.00%	1.000	13
DEU	C3	5.246	0	0.274	0	0.00	1.26%	0.880	25
DEU	C4	0	-1	-0.114	-0.136	-1780.70	-0.39%	1.086	8
DEU	C5	9.495	0	0.101	0	0.00	0.16%	0.931	20
FRA	C6	0	-1	-0.207	-0.254	-3582.70	-0.68%	1.168	5
FRA	C7	7.303	0	0.587	0	14312.37	0.00%	0.758	29
KOR	C8	6.173	0	0.193	0	0.00	0.69%	0.901	24
KOR	C9	0	-0.261	0	-0.009	0.00	-0.02%	1.006	12
KOR	C10	5.777	0	0.386	0	11354.43	0.00%	1.000	13
CAN	C11	0	-0.609	0	-0.03	-1074.92	0.00%	1.023	10
USA	C12	1.376	0	0	0	0.00	0.00%	1.000	13
USA	C13	0	-0.517	0	-0.028	0.00	-0.15%	1.015	11
JPN	C14	0	-1	-0.019	-0.096	-14781.76	-0.40%	1.259	1
JPN	C15	0	-1	-0.115	-0.148	-4191.80	-0.62%	1.086	7
JPN	C16	7.621	0	0.112	0	0.00	0.21%	0.929	21
JPN	C17	43.575	0	0.307	0	0.00	0.42%	0.800	28
JPN	C18	3.401	0	0.313	0	0.00	0.40%	0.810	27
JPN	C19	6.784	0	0.289	0	8374.27	0.00%	0.857	26
JPN	C20	68.87	0	0.27	0	0.00	0.34%	1.000	13
JPN	C21	6.563	0	0.332	0	0.00	0.40%	1.000	13
JPN	C22	16.516	0	0.429	0	0.00	0.48%	0.904	23
SWE	C23	1.277	0	0.118	0	0.00	0.21%	0.931	19
CHN	C24	0	-1	-0.279	-0.312	-3706.10	-1.34%	1.204	3
CHN	C25	0	-1	-0.302	-0.338	-2847.80	-1.06%	1.237	2
CHN	C26	0	-1	-0.252	-0.269	-1328.40	-0.83%	1.187	4
CHN	C27	0	-1	-0.13	-0.141	-746.90	-0.37%	1.094	6
CHN	C28	0	-1	-0.084	-0.095	0.00	-0.20%	1.067	9
CHN	C29	4.67	0	0.133	0	0.00	0.25%	0.918	22
CHN	C30	10.038	0	0.772	0	0.00	1.24%	0.658	30

Figure 3 visually depicts the positional relationship between companies with true fixed-sum outputs and the selected NEEF. Note that the image area is divided into three parts by two dashed lines. The companies in the left part are in smaller sizes, whose fixed-sum outputs are mostly less than the equilibrium state. Besides, they are mainly distributed in JAN, CAN, SWE, and KOR. On the contrary, in the middle part, the majority of companies lie above the NEEF, with four in CHN, one in FRA, one in DEU, and one in JPN. In the right part, these large-scale companies are located near or even on the NEEF, mainly dispersed in USA and DEU except for C14 in JPN. Based upon above, we can conclude that an appropriate increase in inputs can improve the competitive advantage of firms, facilitating them to occupy larger market shares and gain more profits. However, the effect is marginal diminishing. To put it another way, as the size of inputs increases further, the market performance of the firms will not be improved constantly and some inputs are actually redundant.

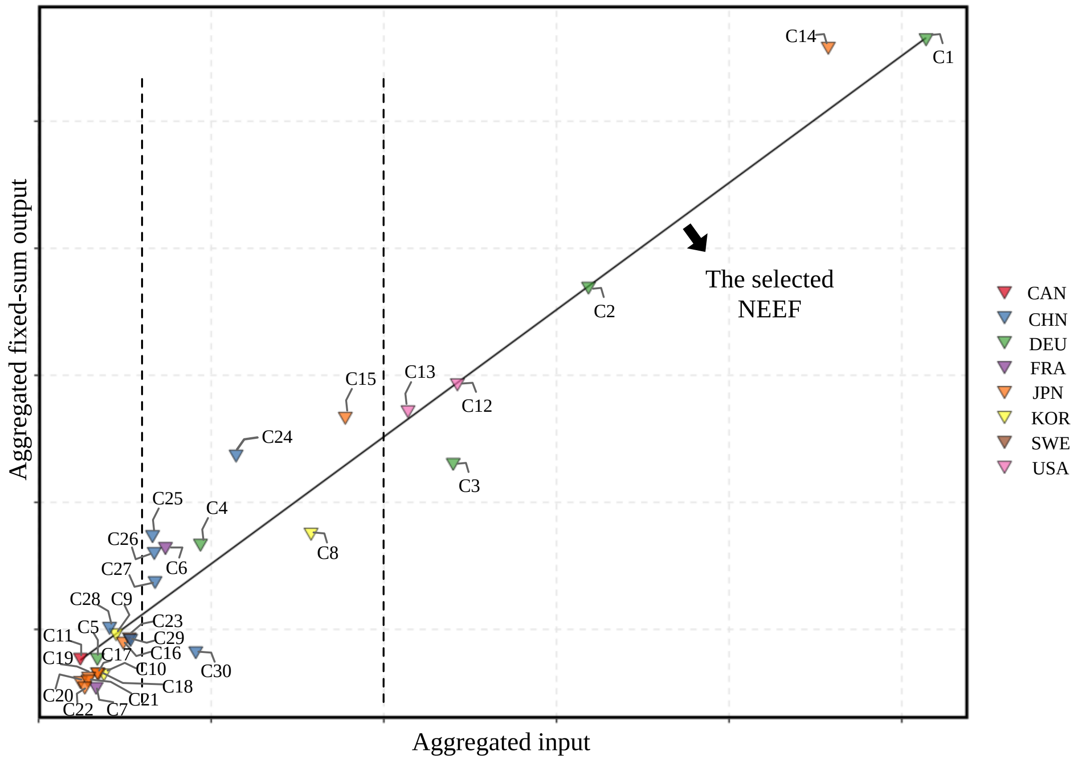

Figure 3. The selected NEEF and the original location of all companies.

Based on the selected NEEF, solve model (6) to assess the non-oriented efficiency of 30 companies, and the results are shown in the last two columns of **Table 5**. From the individual level, C14 is the most efficient, the non-oriented efficiency of which is 1.259. The total assets and employees of C14 rank 2nd and 4th, while its profit and market share rank 1st and 2nd respectively. Therefore, the evaluation result is rational. In addition, C25, C24, and C26 have higher efficiency levels than other companies, due to their strong profitability. These companies with high-efficiency scores are the main engine to lead the whole industry to reach a more ideal equilibrium state. On the contrary, C30, C7, C17, C18, and C19 are in the bottom five among all companies in terms of non-oriented efficiency. They can chase the equilibrium efficiency by reducing inputs and increasing outputs in the proportion of 34.2%, 24.2%, 20%, 19%, and 14.3% respectively.

From the national level, as shown in **Figure 4**, there is a polarization in the performance of Chinese companies, as well as Japanese and French companies. By contrast, companies have narrower efficiency gaps in the USA and KOR. Their non-oriented efficiency is close to the average, denoted by the black shorts in **Figure 4**. Additionally, the average efficiency of the seven Chinese firms is 1.052, which is markedly pulled down by C30, but still ranks first in eight nations. Compared to peers, C30 is too labor-intensive, resulting in the worst performance. The overall performances of companies in CAN and USA rank second and third respectively, with a mean value greater than 1. Note that there is only one firm in SWE, i.e., C23, which ranks 19th among all companies. Except C23, Japanese firms have the poorest average performance. As mentioned before, most Japanese firms are in small size, thus able to enhance market competitiveness through moderately increasing scales. Besides, companies in the FRA, KOR, and DEU have average efficiency of 0.963, 0.969, and 0.979 respectively. Thus, their overall performance is all worse than the equilibrium state.

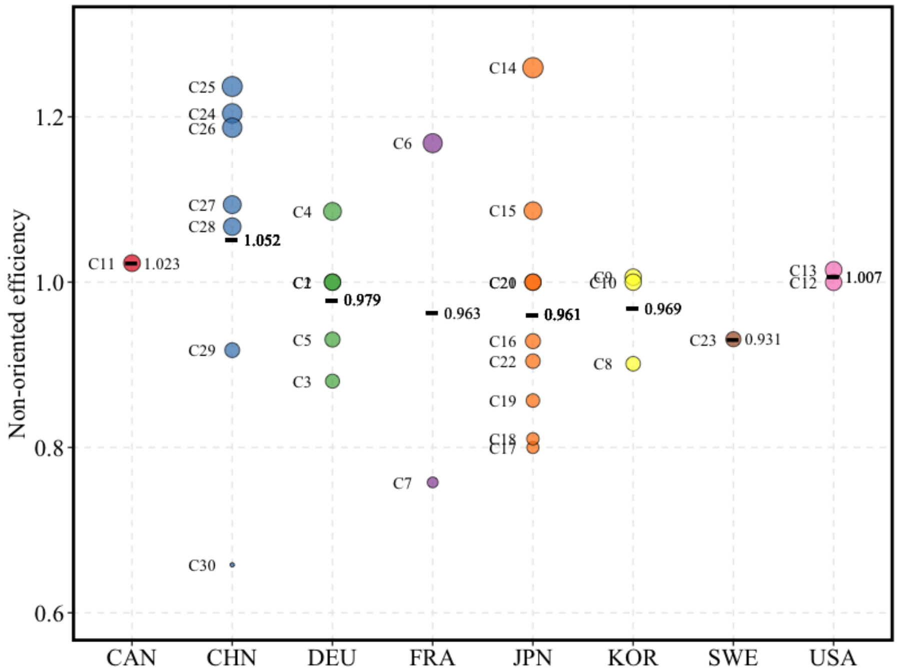

Figure 4. Performance of companies located in eight countries.

Note that the non-oriented efficiency of C10, C20, and C21 is equal to 1, although they actually expand their fixed-sum outputs to achieve the equilibrium efficient state. The phenomenon exposes that our NEEF-based evaluation models have poor discrimination ability for DMUs near the NEEF. This is because NEEFDEA is a radial measurement, which requires inputs to shrink and outputs to expand in the same proportion. Therefore, the efficiency of DMUs will be overestimated when there are some positive slacks in the constraints. To our best knowledge, present equilibrium efficient frontier DEA approaches are all radial measurements. Therefore, some non-radial DEA approaches can be developed to evaluate the efficiency of DMUs with fixed-sum outputs accurately.

4. Summary of Contributions, Managerial Insights and Future Study

This paper proposed the NEEFDEA approach to measure the non-oriented efficiency of DMUs with fixed-sum outputs. Different from prior equilibrium efficient frontier approaches, NEEFDEA takes into account the input-oriented and output-oriented inefficiency simultaneously in both the equilibrium efficient state achieving step and the equilibrium frontier-based evaluation step. In this way, NEEFDEA can provide a more comprehensive evaluation result. However, NEEFDEA always generates either one or an infinite number of NEEFs. Although the positional relationship between each DMU and all optimal NEEFs is stable, the evaluation results of DMUs may vary based on different NEEFs. Therefore, the problem of non-unique NEEFs weakens the effectiveness of NEEFDEA. To address the issue, this paper proposed the bargaining-based selection to pick up a unique and Pareto-optimal NEEF. More specifically, we first calculate the adjustment range of fixed-sum outputs of each DMU to ascertain whether the optimal NEEF is unique. For the cases existing multiple NEEFs, we define the selection process as a Nash bargaining game. DMUs, who have flexibility in adjusting fixed-sum outputs when achieving equilibrium efficient states, are regarded as players. They all seek to select a NEEF to get more beneficial evaluation results. Then we construct the bargaining-based selection model based on the famous Nash bargaining solution to get a unique and Pareto-efficient NEEF. Such NEEF balances the diverse preferences of DMUs and satisfies collective rationality, making the final evaluation result fairer and more acceptable. Next, we use a small data set to illustrate the proposed approaches. By comparing with EEFDEA and GEEFDEA from the previous literature, we summarize the features and improvements of the proposed approaches clearly.

Therefore, our research contributes to the equilibrium efficient frontier DEA approach in the following three respects: Firstly, the proposed NEEFDEA approach extended the prior input-oriented or output-oriented evaluation to a both-oriented evaluation, able to provide more distinguishable and overall efficiency results. Secondly, we suggested an algorithm to clarify a vital issue for this branch of approaches, that is, whether the achieved EEF is unique. Thirdly, the proposed bargaining-based selection approach not only opens up a new way to complete the selection of a unique EEF but also guarantees the Pareto-optimality of the EEF to further enhance the effectiveness of equilibrium frontier approaches.

In addition, from the perspective of practice, the NEEFDEA approach has advantages to help managers to evaluate the performance of rivalry individuals, such as firms in the same industry or participants in the same contest, and so on. Because it can depict the competition of DMUs via introducing proper fixed-sum outputs compared to traditional DEA models. Besides, both the adjustment results of fixed-sum outputs and the evaluation results can provide more valuable information to guide actual operation and management activities. For example, we conduct an empirical study on 30 companies in the vehicle industry in 2020 and obtain the following findings and managerial insights: Firstly, the majority of the 30 companies perform worse than the equilibrium state. Secondly, keeping a moderate scale of employees and assets is beneficial for firms to obtain strong operational performance. Thirdly, the companies distributed in CHN and JPN have polarized efficiency scores, and the average efficiency of Chinese enterprises is the highest while that of Japanese firms is almost the lowest.

Future research can try to propose some non-radial approaches to measure the performance of DMUs with fixed-sum outputs. Because present equilibrium frontier DEA approaches are all radial measurements, which may overestimate the efficiency levels of DMUs. Non-radial measurement can help to get more reliable evaluation results. In addition, our research does not consider the undesirable fixed-sum outputs, which exist widely in the environmental performance evaluation. Therefore, the proposed approach can be extended to deal with undesirable fixed-sum outputs in future.

Reference

[1] Amirteimoori A, Masrouri S, Yang F, Kordrostami S. Context-based competition strategy and performance analysis with fixed-sum outputs: an application to banking sector. J Oper Res Soc 2017;68:1461–9.

[2] Aparicio J, Zofío JL. Economic cross-efficiency. Omega 2021;100:102374.

[3] Bandeira A, Fonseca M, Correia JC, Soares De Mello B, Meza LA. Uniformization of Frontiers in Non-radial ZSG-DEA Models: An Application to Airport Revenues

2010;30:175–93.

[4] Chambers RG, Chung Y, Färe R. Profit, directional distance functions, and Nerlovian efficiency. J Optim Theory Appl 1998;98:351–64.

[5] Charnes A, Cooper WW. Programming with linear fractional functionals. Nav Res Logist Q 1962;9:181–6. https://doi.org/10.1002/nav.3800090303.

[6] Chen L, Jia G. Environmental efficiency analysis of China's regional industry: a data envelopment analysis (DEA) based approach. J Clean Prod 2017;142:846–53. https://doi.org/10.1016/j.jclepro.2016.01.045.

[7] Chen L, Guo M, Li Y, Liang L, Salo A. Efficiency intervals, rank intervals and dominance relations of decision-making units with fixed-sum outputs. Eur J Oper Res 2020. https://doi.org/10.1016/j.ejor.2020.10.024.

[8] Chu J, Wu J, Chu C, Zhang T. DEA-based fixed cost allocation in two-stage systems: Leader-folålower and satisfaction degree bargaining game approaches. Omega 2019. https://doi.org/10.1016/j.omega.2019.03.012.

[9] Dyson RG, Allen R, Camanho AS, Podinovski V V., Sarrico CS, Shale EA. Pitfalls and protocols in DEA. Eur J Oper Res 2001;132:245–59. https://doi.org/10.1016/S0377-2217(00)00149-1.

[10] Fang L. A new approach for achievement of the equilibrium efficient frontier with fixed-sum outputs. J Oper Res Soc 2016;67:412–20. https://doi.org/10.1057/jors.2015.65.

[11] Gomes EG, Lins MPE. Modelling undesirable outputs with zero sum gains data envelopment analysis models. J Oper Res Soc 2008;59:616–23. https://doi.org/10.1057/palgrave.jors.2602384.

[12] Kao C, Liu ST. A slacks-based measure model for calculating cross efficiency in data envelopment analysis. Omega 2020;95:102192. https://doi.org/10.1016/J.OMEGA.2020.102192.

[13] Li Y, Liu J, Ang S, Yang F. Performance evaluation of two-stage network structures with fixed-sum outputs: An application to the 2018winter Olympic Games. Omega 2020. https://doi.org/10.1016/j.omega.2020.102342.

[14] Li Y, Hou W, Zhu W, Li F, Liang L. Provincial carbon emission performance analysis in China based on a Malmquist data envelopment analysis approach with fixed-sum undesirable outputs. Ann Oper Res 2021;304:233–61. https://doi.org/10.1007/s10479-021-04062-8.

[15] Liang L, Wu J, Cook WD, Zhu J. The DEA game cross-efficiency model and its nash equilibrium. Oper Res 2008;56:1278–88. https://doi.org/10.1287/OPRE.1070.0487.

[16] Lins MPE , Gomes EG , Soares de Mello JCCS , Soares de Mello AJRS. Olympic ranking based on a zero sum gains DEA model. Eur J Oper Res 2003;148:312–22. https://doi.org/10.1016/S0377-2217(02)00687-2.

[17] Liu H-H, Song Y-Y, Yang G-L. Interfaces with Other Disciplines Cross-efficiency evaluation in data envelopment analysis based on prospect theory. Eur J Oper Res 2018;273:364–75. https://doi.org/10.1016/j.ejor.2018.07.046.

[18] Lozano S, Hinojosa MA, Mármol AM. Extending the bargaining approach to DEA target setting. Omega 2019;85:94–102. https://doi.org/10.1016/j.omega.2018.05.015.

[19] Nash J. The bargaining problem. Econom J Econom Soc 1950:155–62.

[20] Nash J. Two-person cooperative games. Econom J Econom Soc 1953:128–40.
[21] Ruiz JL. Cross-efficiency evaluation with directional distance functions. Eur J Oper Res 2013;228:181–9. https://doi.org/10.1016/j.ejor.2013.01.030.
[22] Si X, Liang L, Jia G, Yang L, Wu H, Li Y. Proportional sharing and DEA in allocating the fixed cost. Appl Math Comput 2013;219:6580–90. https://doi.org/10.1016/j.amc.2012.12.085.
[23] Wu J, An Q, Yao X, Wang B. Environmental efficiency evaluation of industry in China based on a new fixed sum undesirable output data envelopment analysis. J Clean Prod 2014;74:96–104. https://doi.org/10.1016/j.jclepro.2014.03.054.
[24] Wu J, Xia P, Zhu Q, Chu J. Measuring environmental efficiency of thermoelectric power plants: a common equilibrium efficient frontier DEA approach with fixed-sum undesirable output. Ann Oper Res 2019;275:731–49. https://doi.org/10.1007/s10479-018-2958-0.
[25] Yang F, Wu DD, Liang L, O'Neill L. Competition strategy and efficiency evaluation for decision making units with fixed-sum outputs. Eur J Oper Res 2011;212:560–9. https://doi.org/10.1016/j.ejor.2011.02.029.
[26] Yang M, Li YJ, Liang L. A generalized equilibrium efficient frontier data envelopment analysis approach for evaluating DMUs with fixed-sum outputs. Eur J Oper Res 2015;246:209–17. https://doi.org/10.1016/j.ejor.2015.04.023.
[27] Yang M, Li Y, Chen Y, Liang L. An equilibrium efficiency frontier data envelopment analysis approach for evaluating decision-making units with fixed-sum outputs. Eur J Oper Res 2014;239:479–89. https://doi.org/10.1016/j.ejor.2014.05.013.
[28] Zhu Q, Li X, Li F, Wu J, Zhou D. Energy and environmental efficiency of China's transportation sectors under the constraints of energy consumption and environmental pollutions. Energy Econ 2020;89:104817. https://doi.org/10.1016/j.eneco.2020.104817.
[29] Zhu Q, Li X, Li F, Wu J, Sun J. Analyzing the sustainability of China's industrial sectors: A data-driven approach with total energy consumption constraint. Ecol Indic 2021;122:107235. https://doi.org/10.1016/j.ecolind.2020.107235.
[30] Zhu Q, Song M, Wu J. Extended secondary goal approach for common equilibrium efficient frontier selection in DEA with fixed-sum outputs. Comput Ind Eng 2020;144. https://doi.org/10.1016/j.cie.2020.106483.
[31] Zhu Q, Wu J, Song M, Liang L. A unique equilibrium efficient frontier with fixed-sum outputs in data envelopment analysis. J Oper Res Soc 2017;68:1483–90.