INVESTIGATION OF COVID-19 IMPACT ON TRAFFIC SPEED USING MULTI-LEVEL JOINT MODELS WITH ENDOGENEITY

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Abstract

Speed is considered as a fundamental factor in traffic management and control. Numerous factors have an enormous impact on traffic speed including mandatory traffic policy restrictions. However, there is still lack of conclusive research demonstrating the true impact of COVID-19 on traffic speed for freeways. To fill this gap, the ultimate objective of present study is to investigate the influence of COVID-19 related factors on traffic speed for two multilane highways (I-210 and CA-60). The data were collected from three different sources: Caltrans Freeway Performance Measurement System (PeMS) and Center of Disease Control and Prevention (CDC), which span from February 1, 2020 to April 30, 2020, before and after the implementation of SAH order in the state of California. The study is highlighted with some unique contributions and features. First, multivariate models were utilized to account for the common unobserved heterogeneity shared observational level and hour level for four different lanes. Second, given the strong interdependency between four lanes, endogeneity was explicitly considered. Third, due to the model complexity resulting from multivariate models with the inclusion of endogeneity, the integrated nested Laplace approximation (INLA) algorithm was used over the typical Bayesian hierarchical model based on Markov Chain Monte Carlo (MCMC) approach.

Keywords: Traffic Speed, COVID-19, Speed Prediction Models, Endogeneity.

1. Introduction

Vehicle speed has a huge impact on traffic management and control, and it is considered as an elemental factor to determine the performance of traffic. The estimation of speed distribution of roadway entities could help to improve several traffic-safety programs, planning, and roadway service level. However, the information regarding the speed distribution of road entities is usually unavailable. With that said, the basic relationship of speed with other elemental factors of road entity such as flow or density has been used to estimate the speed (Greenshields, 1935;

Lighthill and Whitham, 1955; Gerlough and Huber, 1976). Based on the relationship between flow, density and speed, numerous studies included several independent variables which could significantly impact the vehicular speed such as weather conditions (Agarwal et al., 2005; Zhao et al., 2012; Ghasemzadeh et al., 2018), driver behavior (Rämä, 1999; Corkle et al., 2001), roadway-built characteristics (Wu et al., 2013; Semeida et al., 2013), traffic characteristics (Vicari et al., 2000; Wang et al., 2007), construction sites (Kang et al., 2004; Weng and Meng, 2011), and so on. Among them, a subset of studies incorporated special events as the explanatory variable to investigate the influence on traffic speed.

Special events such as sports matches, organized gatherings, fairs, disasters, or implementation of restrictive traffic policies could potentially impact the traffic behavior (Wojtowicz and Wallace, 2010; Kwoczek et al., 2014; Tempelmeier et al., 2020). As is well known, due to the rapid spread of the novel coronavirus (COVID-19), the stay-at home (SAH) order had been implemented in the state of California on March 19, 2020. The consequences of SAH directive have greatly influenced various domains including economy (Fernandes, 2020; Ozili and Arun, 2020), public healthcare sector (Ji et al., 2020; Tanne et al., 2020), tourism and aviation field (Abu-Rayash and Dincer, 2020; Nicola et al., 2020), educational realm (Crawford et al., 2020; Sintema, 2020), transportation field (Kerimray et al., 2020; Huang et al., 2020), and many more. Even though a plethora of studies have been extensively dedicated to different types of special events, there is still alack of conclusive research results regarding the investigation of COVID-19 impact on roadway traffic speed. Therefore, incorporating COVID-19 impact on traffic speed into traffic characteristics would facilitate the development of better strategies and policies to engender a safe environment for all roadway users.

Given such context, there has been considerable interest in developing speed prediction models to obtain crucial insights of mobility behavior. For instance, Pei et al. (2012) developed the prediction model for speed distribution by employing Full Bayesian method and considered the effects of weather conditions, road geometry, and traffic flow. Another study conducted by Silvano and Bang (2015) dedicated on the effect of posted speed limit on urban roads by including the variables pertaining to roadway-built characteristics such as presence on-street parking and sidewalks, road environment, and carriageway width.

In addition to the above studies which were focused on the development of speed prediction models, some studies have utilized endogeneity models to account for the simultaneity issues (Eisenberg, 2003; Washington et al., 2010; Cheng et al., 2018) between the explanatory and response variables. Previous safety literature shows that large number of studies developed the models by assuming the unidirectional relationship between independent and dependent variables, in which only independent variables could affect the dependent variables. Nonetheless, few studies in the past revealed different simultaneity issue where the phenomenon could be inverse, which means output variables could also influence the input variables. Ignorance of the issue of endogeneity could lead to bias and unreliable inferences (Mannering and Bhat, 2014). To overcome this problem, some studies attempted to address the endogeneity problem by employing different methods (Dane et al., 2014). For example, Himes and Donnell (2010) developed mean operating speed model and speed deviation model by employing three-stage least squares (3SLS) estimator to investigate the traffic flow and roadway-built factors with the consideration of endogeneity.

Building upon the previous studies, the main objective of present study is to investigate the impact of COVID-19 on traffic speed. For this purpose, the data were collected from three unique sources: Caltrans Freeway Performance Measurement System (PeMS) to obtain the traffic information of two parallel freeway (I-210 and CA-60) situated within the Los Angeles County in California; Center of Disease Control and Prevention (CDC) to collect the number of daily confirmed COVID-19 cases; and incorporation of data span February 1st to April 30th before and after the implementation of SAH order in the state of California. First, multivariate models were utilized to account for the common unobserved heterogeneity shared observational level and hour level for four different lanes. Second, given the strong interdependency between four lanes, endogeneity was explicitly considered. Third, two sample t-test, Fisher's F-Test and Welch's T-Test were used to determine the SAH order on traffic speed. Fourth, due to the model complexity resulting from multivariate models with the inclusion of endogeneity, the integrated nested Laplace approximation (INLA) algorithm was used over the typical Bayesian hierarchical model based on Markov Chain Monte Carlo (MCMC) approach. Finally, to assess the performance of models, various evaluation criteria were employed including, Deviance Information Criteria (DIC), Watanabe-Akaike Information Criteria (WAIC), and Log pseudo Marginal Likelihood (LPML).

DATA DESCRIPTION

The present study collected the data from three unique sources. The real-time traffic characteristic information, such as the speed and volume, was obtained from the Caltrans' PeMS website (www.pems.dot.ca.gov, 2020). The data are collected via loop stations located the freeway(s) at specific locations including on-ramps, off-ramps, and along the freeway itself. To limit the potential recordings of duplicate vehicles resulting from the weaving actions, the authors took great care to utilize data collected from stations that are located further away from on-, off-ramps as well as freeway interchanges. (Golob et al., 2004), which could ensure a more stable traffic volume and speed from the basic freeway segments. For illustration purpose, the two parallel freeways (east to west) in which data were collected consist of the I-210 and the CA-60 freeways within the Los Angeles County of District 7 in California. Overall, 56 loop stations were selected from the three freeways that contain 4 lanes in each direction. These stations go through a wide range of land-use types such as commercial, industrial, and residential which provide extensive volume variations.

As mentioned in the introduction section, the weather situation is also an important factor that greatly influences traffic speed. It is for this reason why the authors chose to include various weather conditions into the models. The weather information was collected from Weather Underground (www.weatherunderground.com) that included data such as Temperature, Humidity, Windspeed, Pressure, and Total Precipitation. Additionally, data regarding the daily confirmed cases of Covid-19 is hypothesized to have an impact on traffic volume due to those being tested for Covid-19 would have to visit a testing center or hospital to confirm their diagnosis. The daily confirmed cases of Covid-19 in Los Angeles County were collected from the Centers for Disease Control and Prevention (CDC) website (www.cdc.gov). A summary of all variables used within the analysis can be found in Table 1.

Table 1. Descriptive Statistics of Collected Data

Variable	Description	Mean (SD)	Min	Max
Lane 1 Average Speed (L.S.1)	Average Hourly Vehicles Speed in Each Lane (mph)	69.94 (9.17)	4.3	78.9
Lane 2 Average Speed (L.S.2)		65.25 (8.58)	3.5	77.5
Lane 3 Average Speed (L.S.3)		58.38 (9.65)	4.4	77.2
Lane 4 Average Speed (L.S.4)		57.58 (8.50)	10.2	72.5
Total Flow	Hourly Traffic Flow	3240.32 (1864.15)	217	7308
Time	The count of hours for each day	12.5 (6.92)	1	24
Days	The count of days from February 1st to April 30th	45.5 (25.98)	1	90
Daily Confirmed	The number of COVID-19 new infections in Los Angeles County for each day	258 (365.69)	0	1509
Temperature	The average temperature of the day in degrees Fahrenheit	59.38 (8.46)	35	97
CA-60	Two freeways in Los Angeles County	2158 (50.0%)		
I-210		2158 (50.0%)		
SAH0	Stay at Home policy issued on March 19th ;0 represents the days before the policy was issued; 1 represents the days after the policy was issued	2252 (52.2%)		
SAH1		2064 (47.8%)		

Note: SD represents Standard Deviation; Min represents minimum; Max represents maximum.

In addition to the COVID-19 confirmed case number and pertinent policy like SAH, the paper also considers the weather-related info and its impact on the speed. Amongst a set of relative factors, only the temperature data were collected due to two reasons. First, the temperature has been shown in previous literature to exert strong influence on the speed (Nasimifar et al., 2018; Bodin et al., 2016; Greenfield et al., 2012; Lin et al., 2015). Second, other important info such as precipitation, vision, and dew point etc. are pretty stable at the study time period in southern California due to its unique geographical nature. Such near-zero-variance variables were excluded from the model development since they have less predictive power and tend to cause a model to crash (Kuhn, M., & Johnson, K. (2013). It is the hope of authors that the employment of temperature as the weather representative variable can capture the influence of other related covariates to some degree as well.

3. METHODOLOGY

As previously mentioned, the primary goal of this study is to evaluate the impact of COVID-19 and the related factors such as SAH policy, the number of COVID-19 cases per day, and the weather conditions on the freeway traffic speed. For this purpose, a two-sample t-test was first performed to determine the significance of SAH order on traffic speed. Second, under the framework of INLA, a multivariate joint model with endogeneity was developed to determine the impact of associated factors on traffic speed. Third, to evaluate the model performance, distinct goodness-of-fit assessment measures were adopted. The details of each section are shown below.

3.1. Model Specification

To develop models of four lanes of both freeways (CA-60 and I-120), few independent variables expected to impact one or all the four lanes were incorporated. This study employed multivariate framework with the Gaussian distribution as shown below:

$$y = \beta_0 + \boldsymbol{\beta X} + \varepsilon \tag{1}$$

Where y represents a matrix consisting of lane-mean speed of four lanes, and β_0 represents a global intercept vector for four different lanes. $\boldsymbol{\beta}$ is the regression coefficient vector, **X** is the covariate matrix, and ε represents the white noise. To better understand the multivariate models with endogeneity and different covariates, Equation 4 can be further expanded to the following expressions:

$$y_{L_1} = \beta_{0L_1} + \beta_{L_1}\boldsymbol{X} + \beta_{L_2}y_{L_2} + \varepsilon_{OL_1} + \varepsilon_{HL_1} + \beta_{SAH_1} * SAH + \beta_{DC_1} * DC + \beta_{Day_1} * Day + \beta_{SAH*DC_1} * SAH * DC + \beta_{SAH*Day_1} * SAH * Day \tag{2}$$

$$y_{L_2} = \beta_{0L_2} + \beta_{L_2}\boldsymbol{X} + \beta_{L_1}y_{L_1} + \beta_{L_3}y_{L_3} + \varepsilon_{OL_2} + \varepsilon_{HL_2} + \beta_{SAH_2} * SAH + \beta_{DC_2} * DC + \beta_{Day_2} * Day + \beta_{SAH*DC_2} * SAH * DC + \beta_{SAH*Day_2} * SAH * Day \tag{3}$$

$$y_{L_3} = \beta_{0L_3} + \beta_{L_3}\boldsymbol{X} + \beta_{L_2}y_{L_2} + \beta_{L_4}y_{L_4} + \varepsilon_{OL_3} + \varepsilon_{HL_3} + \beta_{SAH_3} * SAH + \beta_{DC_3} * DC + \beta_{Day_3} * Day + \beta_{SAH*DC_3} * SAH * DC + \beta_{SAH*Day_3} * SAH * Day \quad (4)$$

$$y_{L_4} = \beta_{0L_4} + \beta_{L_4}\boldsymbol{X} + \beta_{L_3}y_{L_3} + \varepsilon_{OL_4} + \varepsilon_{HL_4} + \beta_{SAH_4} * SAH + \beta_{DC_4} * DC + \beta_{Day_4} * Day + \beta_{SAH*DC_4} * SAH * DC + \beta_{SAH*Day_4} * SAH * Day \quad (5)$$

Where subscripts L_1, L_2, L_3, L_4 represents four different lanes of freeway, β_0 is the global intercept, β is the regression coefficient vector, **X** is the covariate matrix, SAH is stay at home, DC is daily confirmed cases of COVID-19, and ε represents the white noise or random effects. The presence of endogeneity can be denoted by the statistical significance of $\beta_{L_1}, \beta_{L_2}, \beta_{L_3}$ and β_{L_4}.

In multivariate framework, both random effects of observational ($\varepsilon_{OL_1}, \varepsilon_{OL_2}, \varepsilon_{OL_3}, \varepsilon_{OL_4}$) and hour levels ($\varepsilon_{HL_1}, \varepsilon_{HL_2}, \varepsilon_{HL_3}, \varepsilon_{HL_4}$) follow a multivariate normal distribution. The hierarchical process for both levels can be presented as follows:

$$\varepsilon_j \sim Normal\ (\mu, \Sigma) \quad (6)$$

Where

$$\varepsilon_j = \begin{pmatrix} \boldsymbol{\varepsilon}_{L1} \\ \boldsymbol{\varepsilon}_{L2} \\ \boldsymbol{\varepsilon}_{L3} \\ \boldsymbol{\varepsilon}_{L4} \end{pmatrix}, \mu = \begin{pmatrix} \mu_{L1} \\ \mu_{L2} \\ \mu_{L3} \\ \mu_{L4} \end{pmatrix}, \Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{pmatrix} \quad (7)$$

In above equations, ε_j is the independent random effect which captures the extra-Poisson heterogeneity among observation or hour levels, μ is the vector of mean values for four different lanes, $\sum$ is the variance-covariance matrix. The inverse of the variance-covariance matrix represents the precision matrix and can be represented by:

$$\Sigma^{-1} \sim Wishart(I, J) \quad (8)$$

Where I is the identity matrix of J×J with J is degree of freedom (Congdon, 2006).

3.2. Two Sample T-Test

To determine if the SAH order has a significant effect on traffic speed, a two-sample t-test is conducted. To determine which t-test will be better implemented, Fisher's F-Test is needed to first determine the homogeneity of variance (Box, 1953) between the traffic speed prior to and following the SAH order. The results of the F-Test will determine which T-Test would be superior with the given data.

3.2.1. Fisher's F-Test

Fisher's F-Test computes the ratio between the explained variance, in this case would be the traffic speed(s) before the SAH order was issued, and the unexplained variance, or the traffic speed(s) after the SAH order being issued. The F-Test is performed with the following equation (Armitage and Berry, 2001):

$$F = \frac{S_b^2}{S_a^2} \tag{9}$$

Where S^2_b represents the explained variance and S^2_a represents the unexplained variance. The F-value calculated with Equation (1) is compared to the corresponding F-Table's value unexplained variance. If the F-Table's value of S^2_b is less than the calculated F-value from Equation (1), then it is necessary reject the null hypothesis. In this study, the null hypothesis refers to the SAH order having a significant effect on traffic speed.

3.2.2 Welch's Two-Sample T-Test

There are two well-known methods for performing a two-sample t-test. The first is known as the Classical t-test, also known as the Student's T-Test, and is used primarily between samples whose variances are equal (Efron, B., 1969). The Second is Welch's T-Test, contrary to the Classical T-Test, Welch's T-Test performs better with samples whose variances are unequal (Welch, 1947). Welch's T-Test involves the generating a t-value and the degrees of freedom as shown below:

$$t = \frac{m_b - m_a}{\sqrt{\frac{S_b^2}{n_b} + \frac{S_a^2}{n_a}}} \tag{10}$$

$$df = \left(\frac{S_b^2}{n_b} + \frac{S_a^2}{n_a}\right) \Big/ \left(\frac{S_b^2}{n_b^2(n_b-1)} + \frac{S_a^2}{n_a^2(n_a-1)}\right) \tag{11}$$

Where m_b and m_a represent the sample means of traffic speed before and after the SAH order was issued respectively. The samples sizes before (n_b) and after (n_a) are also utilized in both equation (2) and (3). To determine if the results from Welch's T-Test are effective, the degrees of freedom between the two variances must be greater than 5 (Allwood, 2008). Additionally, the predictive value (p-value) is also calculated to act as another form of verification in determining if the SAH order has a significant impact on traffic speed.

3.3 Evaluation Criteria

Various criteria employed in the present study for assessment of predictive accuracy and goodness-of-fit are illustrated in the following sections.

3.3.1. Deviance Information Criteria (DIC)

DIC (Deviance Information Criteria) is a Bayesian generalization of Akaike Information Criteria (AIC) () used to evaluate the complexity and goodness-of-fit of the models. This criterion was developed by Spiegelhalter (2002) and can be written as:

$$DIC = (\overline{D}) + P_D \tag{12}$$

Where $(\overline{D})$ represents the posterior mean deviance and P_D represents the effective coefficient number. The models which display a larger DIC value (higher than +7) is less efficient in its abilities to accurately predict additional data (Spiegelhalter et al., 2002).

3.3.2 Watanabe-Akaike Information Criteria (WAIC)

WAIC is another heavily used criteria to determine the efficiency of hierarchical models. As mentioned before, WAIC, like DIC, is derived from AIC with the primary difference being that WAIC employs a posterior distribution rather than a point estimation (Watanabe, S., 2013). This allows WAIC to be a more convenient approximation for cross-validation since it uses In-Of-Sample data (Aregay et al., 2017). WAIC is based on pointwise predictive density (Watanabe, S., 2013) which is set on a logarithmic scale that is commonly used as the baseline value to estimate predicted density (Gelman, A. et al., 2014). The Log Pointwise Predictive Density (LLPD) is calculated with the following equation:

$$LLPD = \sum_{i=1}^{n} \log\left(\frac{1}{S}\sum_{s=1}^{S} p(y_i|\theta^s)\right) \tag{13}$$

Through the assumption that the number of simulations draws S is large enough to completely encase the posterior distribution, only then is it possible to allow for the interchangeability between the LLPD and the computed LLPD of the data from the Equation (13). It is for this reason that this criterion is similar to DIC in how it, explains model efficiency through a lower LLPD value calculated with each model (Gelman et al., 2014).

3.3.3 Log pseudo Marginal Likelihood (LPML)

The final criterion, LMPL, originally proposed by Geisser and Eddy in 1979, it has been widely utilized in numerous studies (Gelfand and Mallick, 1995; Zhao and Hanson, 2011). Unlike the previously mentioned model efficiency criteria, LPML performs cross-validation using OOS data via leave one out method. LPML is calculated based on the Conditional Predictive Ordinate (CPO) which is primarily used to diagnose and select models (Muthukumarana and Tiwari, 2016) is can be calculated through the equation (Zhang et al, 2017):

$$CPO_i = \int f(y_i|\theta, x_i)\pi\left(\theta|D^{(-i)}\right)d\theta \tag{14}$$

In this equation, θ is the unknown parameter of interest; y_i and x_i are the response and covariate vectors; $D^{(-i)}$ is the data set without the ith observation and $\pi(\theta| D^{(-i)})$ is the posterior density of θ based on data $D^{(-i)}$. LPML is defined as:

$$LPML = \sum_{i=1}^{n} \log(CPO_i) \quad (15)$$

Unlike the models derived from AIC, LPML represents models with higher efficiencies and capabilities to predict additional data accurately with lower values. Besides, by utilizing OOS data for cross-validation, there isn't a need to apply a penalty for the number of models generated to take data bias into account (Jiang et al., 2016).

4. Results

The observed homogeneity between the variances of traffic speed before and after the SAH order was issued was first performed by Fisher's F-Test, and then verified by Welch's T-Test are displayed to illustrate the direct impact the SAH order itself had on the overall traffic speed among the two previously mentioned freeways. Additionally, the multivariate-joint model to outline the effects that SAH order as well as the aforementioned variables outlined in Table 1 is supplied. Lastly, each model generated, one for each freeway, is evaluated using the previously mentioned model evaluation criteria and the results are presented.

4.1 Two-Sample T-test

Prior to using the T-Test to understand the direct effects that the SAH order had on traffic speed, it is necessary to first perform Fisher's F-Test. As mentioned in the Methodology, based on the results from Fisher's F-Test, the proper T-Test can then be utilized to demonstrate the effects of the SAH order on traffic speed. The values resulting from Fisher's F-Test are shown in Table 2.

Table 2. F-test for Homogeneity in Variances

Freeway	**I-10**				**I-210**			
Lane Speed	LS.1	LS.2	LS.3	LS.4	LS.1	LS.2	LS.3	LS.4
F	8.36	6.44	1.96	2.51	6.86	2.61	1.36	6.33
DF (0)	1125	1125	1125	1125	1125	1125	1125	1125
DF (1)	1031	1031	1031	1031	1031	1031	1031	1031
P-values	**<2e-16**	**<2e-16**	**<2e-16**	**<2e-16**	**<2e-16**	**<2e-16**	**5.3e-07**	**<2e-16**

Notes: 1. F values were derived from equation (1).
2. Here "DF (0)" indicates the degree of freedom of the observations before the SAH order was issued, and "DF (1)" represents the observations after the SAH order was issued.
3. Statistically significant variables with p-value less than 0.05 were shown in font bold, and these tested variables reject the null hypothesis. The table shows that there is a significant difference in the variance of traffic flow before and after the SAH policy was established.

As shown in Table 2 are the F-values, p-values, and the degree of freedom (DF) between traffic speed before and after the SAH order was issued. In addition to the F-values for both freeways, the variance of the total traffic speed is calculated and then compared with the F-value. As mentioned in the Methodology, this is to ensure that there is a significant change in traffic speed based on whether or not the null hypothesis is rejected. The predictive values (p-values) are all less than 0.05 which indicates that each prediction of traffic speed within each freeway and lane falls within a 95% confidence interval (Wasserstein, R. L. & Lazar, N. A., 2016). Due to the variance of the traffic speed before and after the SAH order, as seen in Table 2 with each F-value not equaling to 1, the appropriate T-Test to perform is Welch's T-Test, whose results are presented in Table 3.

Table 3. Welch's Two-Sample T-Test Outcomes

Freeway	**I-10**				**I-210**			
Lane Speed	LS.1	LS.2	LS.3	LS.4	LS.1	LS.2	LS.3	LS.4
t	-18.6	-14.71	-5.75	-5.87	-12.6	-7.99	0.31	-7.4
DF	1411.66	1491.55	2037.93	1919.59	1470.88	1899.34	2146.64	1497.73
P-values	**<2e-16**	**<2e-16**	**1.0e-08**	**5.1e-09**	**<2e-16**	**2.3e-16**	0.76	**2.3e-13**

Notes: 1. The t values were derived from equation (**2**) and equation (**3**).
2. The "DF" stands for the degree of freedom of the hourly vehicle speed observations.

Similar to the results illustrated in Table 2, Table 3 also summarizes the DF, t-values, and p-values that resulted from Welch's t-Test. As seen in Table 3, the DF of both freeways and all lanes involved are all greater than 5 which indicates the effectiveness of the T-Test. The presented p-values are, again, have the same criteria for statistical significance to land within the 95% confidence interval as mentioned in Table 2. In addition, the p-values of all lanes from both freeways with the exception of lane 3 from the I-210 freeway. This could be explained as different lanes could have different purposes as explored by Senathipathi et al. (2010). To facilitate the readers, the frequency of traffic speed is visualized by using box and whisker's plot as shown in Figure 1.

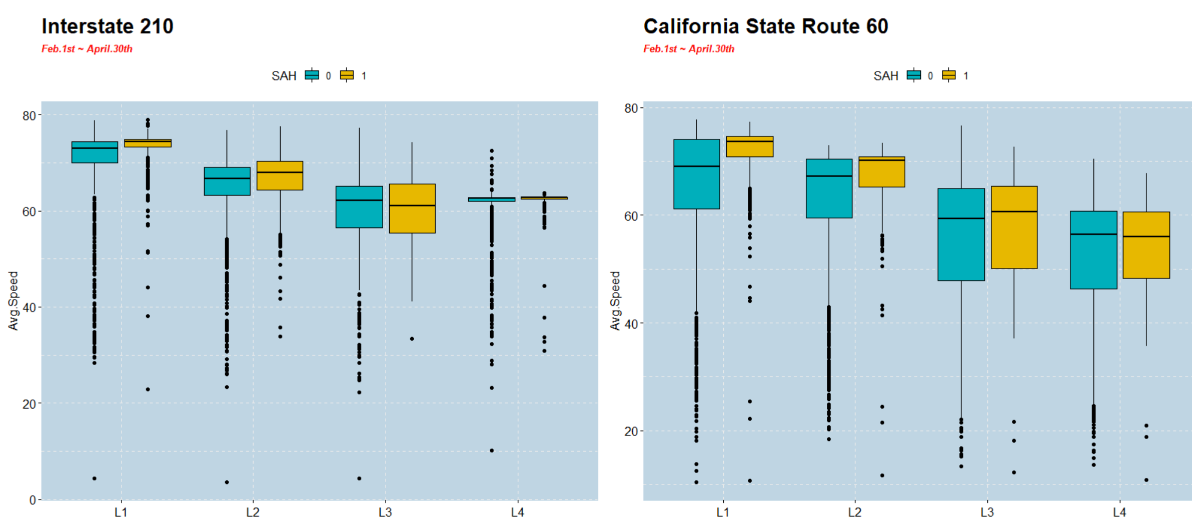

Figure 1. Average Vehicles Speed on Each Lane of 2 Highway Stations Before and After the SAH Policy from February 1st to April 30th.

The box and whisker's plot in Figure 1, showcases the frequencies of traffic speed of vehicles recorded for each lane within both freeways in terms of the first quartile, mean, third quartile as well as one standard deviation both before and after the first and third quartiles, respectively. The statistical outliers are represented as individual dots along the tails created by the aforementioned standard deviations. The traffic speeds before the SAH order are represented as the teal box on the left, with the legend representing it as SAH0. Likewise, the traffic speeds recorded after the SAH order are in yellow and are represented as SAH1 within the legend. The y-axis represents the average speed recorded for that day.

Upon closer inspection of Figure 1, there are clear differences between the traffic speeds before and after the SAH order was given within lanes 1 and 2 from each freeway. Although lanes 3 and 4 do have statistical differences as seen in Table 3, except for lane 3 from the I-210 freeway, these differences are less apparent. Another apparent aspect is the differences in traffic speeds within each lane. This solidifies the previous statement following Table 3 and is confirmed from previous literature (Hurdle et al., 1997; Jiang, 1999).

4.2 Model Estimates

Table 4. Summary of the Four Lanes Speed Joint Model

	CA-60				I-210			
Variable	β (LS.1) **Mean (SD)**	β (LS.2) **Mean (SD)**	β (LS.3) **Mean (SD)**	β (LS.4) **Mean (SD)**	β (LS.1) **Mean (SD)**	β (LS.2) **Mean (SD)**	β (LS.3) **Mean (SD)**	β (LS.4) **Mean (SD)**
	Fixed Effects							
Intercept	-0.002 (0.949)	**3.1 (1.192)**	**-3.227 (1.413)**	**7.765 (1.306)**	**8.946 (1.148)**	**-3.868 (1.704)**	**22.821 (2.33)**	**37.085 (1.67)**
Total Flow	**2.537 (0.393)**	-0.475 (0.338)	**-3.397 (0.503)**	**3.687 (0.435)**	**-4.867 (0.453)**	**2.703 (0.545)**	**4.316 (0.835)**	**-16.372 (0.57)**
SAH	**2.54 (0.574)**	**1.098 (0.45)**	0.696 (0.687)	**-2.728 (0.569)**	**8.644 (0.638)**	**-1.907 (0.727)**	**-22.748 (0.929)**	**7.723 (0.601)**
Day	**-0.017 (0.007)**	**0.024 (0.005)**	-0.013 (0.008)	0.01 (0.007)	**0.018 (0.007)**	**-0.025 (0.007)**	-0.017 (0.01)	**0.02 (0.006)**
Daily Confirmed	**63.245 (11.902)**	**-20.089 (9.768)**	25.456 (13.164)	**-33.959 (11.779)**	**55.782 (12.274)**	5.982 (12.394)	**-93.12 (15.434)**	**24.298 (11.011)**
SAH * Daily Confirmed	**-62.639 (11.915)**	**20.302 (9.775)**	**-27.303 (13.174)**	**35.188 (11.791)**	**-53.644 (12.283)**	-6.828 (12.391)	**89.649 (15.447)**	**-22.397 (11.015)**
SAH * Day	0.011 (0.011)	**-0.035 (0.009)**	-0.006 (0.013)	**0.038 (0.011)**	**-0.141 (0.012)**	**0.051 (0.013)**	**0.344 (0.018)**	**-0.149 (0.011)**
Temperature	**-2.205 (0.641)**	-0.02 (0.491)	**1.357 (0.729)**	**-2.245 (0.628)**	**4.006 (0.578)**	-0.799 (0.584)	**-4.922 (0.819)**	**2.47 (0.491)**
LS.1		**0.655 (0.009)**				**0.787 (0.014)**		
LS.2	**0.991 (0.007)**		**0.174 (0.019)**		**0.821 (0.01)**		**0.45 (0.021)**	
LS.3		**0.279 (0.008)**		**0.828 (0.006)**		**0.244 (0.013)**		**0.314 (0.01)**
LS.4			**0.931 (0.019)**				**0.517 (0.035)**	
	Random Effects							
Observation. ID	**19.941 (3.972)**	**26.692 (6.803)**	**27.464 (6.014)**	**10.743 (4.869)**	**18.412 (5.267)**	**22.895 (6.387)**	**23.882 (5.887)**	**11.752 (4.814)**
Hour.ID	**0.714 (0.248)**	**1.979 (0.707)**	**0.494 (0.405)**	**1.406 (0.586)**	**0.304 (0.105)**	**0.488 (0.152)**	**0.153 (0.047)**	**0.248 (0.074)**

	Goodness-of-fit Criteria	
DIC	40163.98	42437.01
WAIC	40199.17	42502.98
LPML	-20099.44	-21253.67

Notes: 1. SD refers to Standard Deviation.
2. The estimates in bold font represent the variables that have a significant impact on lane speed at a 95% level.

Upon detailed inspection of Table 4, the first thing that stands out was the statistical significance of the endogeneity show by the immediate adjacent lanes. As seen in Table 4, it is clear that any increase in traffic speed also influences its adjacent lanes in which it also increases their speed as well. Moridpour et al. (2010) performed a study to understand the effects of similar endogenic variables. One of their conclusions supports the findings of Table 4 in that the endogenic variables are significant and that any increase in speed in one lane will similarly influence its adjacent lanes. In addition to the mentioned fixed variables that are significant among all lanes within both freeways, the random variables that consist of the observation id, as well as the hour id, are also statistically significant among all lanes presented.

Among each freeway presented, it is clear that different variables are considered significant within specific lanes. This can be explained that in addition to each lane having a specific purpose but the freeways, despite being within the same county, are located in different areas within the county and therefore will have a different population with different needs. This is apparent with Temperature, as the temperature increases, there is both an increase in specific lanes as well as a decrease in traffic speed in others. This same effect is present in other variables such as the implementation of the SAH order, the Daily confirmed, as well as total flow to name a few.

The model efficiency criteria utilized in this study do produce interesting results, both DIC and WAIC scores for each freeway are similar to one another. While this is expected due to both of these criteria being derived from AIC, it does aid in justifying the use of random parameters as well as the use of endogenic variables within the model to ensure model prediction accuracy (Vehtari et al., 2015). This also solidifies the idea of multivariate-joint models being capable of producing accurate results that are more complex than their simple fixed parameter counterparts (Pettitt et al., 2002; Hickey et al., 2018).

In addition to understanding the endogenic variables within each lane and its effects on one another, it is also important to understand the overall correlation and covariance between each lane. Table 5 highlights the correlation and covariance between each lane within both freeways.

Table 5: Correlation and Covariance Matrix Between the Coefficients of Different Lanes' Unobserved Heterogeneity

Observation.ID	**CA-60**				**I-210**			
	β (LS.1)	**β (LS.2)**	**β (LS.3)**	**β (LS.4)**	**β (LS.1)**	**β (LS.2)**	**β (LS.3)**	**β (LS.4)**
β (LS.1)	1.00 1.4e-04	0.28	0.00	0.15	1.00 4.5e-04	-0.05	0.01	**-0.86**
β (LS.2)	2.2e-05	1.00	0.00	**-0.70**	-1.2e-05	1.00	0.01	0.20

		4.2e-05				1.3e-04		
β (LS.3)	3.5e-09	1.2e-09	1.00 3.2e-05	**-0.61**	2.1e-06	1.3e-06	1.00 1.0e-04	-0.20
β (LS.4)	4.2e-11	-1.1e-10	-8.1e-11	1.00 5.5e-16	-6.3e-06	7.9e-07	-7.0e-07	1.00 1.2e-07

Notes: 1. The upper triangle of the matrix shows the correlation coefficients of the estimates of Observation ID, and the gray grids in the lower triangle of the matrix show the covariance. Correlation coefficients and covariances are listed in the diagonal grids of the matrix at the same time.
2. Highly correlated estimates with correlation coefficients greater than 0.6 are marked as a bold font.

Table 5 shows both the correlation and the covariance that each lane has on one another within each freeway. The upper left portion of each freeway's matrix shows the correlation between two lanes and those values whose absolute value is greater than 0.6 are considered highly correlated and are bolded. The covariances between any two lanes are located in the lower-left matrix with the gray filling.

Upon review of Table 5, it is clear that lane 1 from the CA-60 freeway is not highly correlated with any other lane however lanes 2 and 3 are highly correlated with lane 4. More specifically these lanes are negatively correlated which contradicts the findings from Table 4. Additionally, lane 1 is highly correlated with lane 4, again in a negative light, which isn't adjacent. This suggests that in addition to immediate adjacent lanes affecting one another, further lanes could also affect one another.

5. Conclusions

This study performs a multivariate model in efforts to understand the impact of Covid-19 on traffic speed, more specifically the impact that the SAH order as well as the daily confirmed cases have on traffic speed. The data utilized were obtained from the PeMS website regarding the CA-60 and I-210 freeways in Los Angeles County, the daily confirmed cases were collected from the CDC website and the weather data originated from the weather underground. The observation dates range from February 1, 2020 to April 30, 2020. Based upon the results, conclusions are drawn as follows:

1. It is rather unclear that the SAH order was a direct cause of higher traffic speeds as it is clear that the implementation of the SAH order has both increased and decreased the traffic speed depending on the lane.
2. There is a clear effect that the traffic speed from one lane affects its immediate neighboring lanes. However, newer evidence suggests that traffic speed from one lane may also affect more than just the nearest proximity lane.
3. With the given data and variables utilized in this study, it is clear that using both endogenic variables and random variables improve upon the model prediction accuracy due to their statistical significance.

In addition to the aforementioned findings, there are few caveats required in this study. First, weather related variables such as wind speed and precipitation have been included in numerous amounts of literature and would be beneficial to include such variables in future research.

Second, generating a new model that includes the effect of the endogeneity produced by non-adjacent lanes. This will ensure a more complex model and could potentially alter the predictive capabilities of this modeling method. Third, the utilization of other multivariate-joint modeling methods such as the use of a MCMC could provide a more thorough analysis due to its high accuracy rate with the only drawback being the computation time. Fourth, in addition to performing a space-time analysis of traffic speed, an analysis regarding the impact of Covid-19 on traffic volume, traffic accident frequency as well as accident severity could only prove to be beneficial to society not only for today, amidst the pandemic, but benefit future policies as well.

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