USING NONCONVEX OPTIMIZATION METHOD TO MODEL COMPLEX ASSET ALLOCATION AND LOCATION PROBLEMS

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ABSTRACT

Policy decisions regarding allocating goods, selecting locations, and determining storage facility types and capacities often employ conventional optimization methodologies such as linear or nonlinear programming. This prevailing approach is partly grounded in the assumption of a consensus among diverse decision-makers, consequently favoring the adoption of a singular objective or a hierarchy of ranked objectives. Nevertheless, certain scenarios warrant a departure from this norm. Particularly within specific semi-hierarchical contexts, the influence of one group of decision-makers can subtly constrain another group. Such circumstances manifest notably in problems that align with the Stackelberg leader-follower game in the realm of policy challenges. In the present study, we introduce an empirical quandary concerning the allocation and positioning of real-world assets. We have conceptualized and effectively addressed this issue as a bilevel programming problem, meticulously capturing the intricate web of decision trade-offs that it entails.

Keywords: Bilevel Optimization; Global Optimization; Asset Allocation; Location Problem; Stackelberg Game.

INTRODUCTION

A prominent international corporation is evaluating the distribution of its resources across its expansive and diverse array of facilities (with 14 existing and six potential locations). The central planners at the corporate headquarters aspire to enhance the overall system by minimizing the aggregate cost of global operations. Their objective is to identify operational efficiencies that align with the corporation's long-term strategy, particularly as it extends its presence into novel territories. To put it succinctly, the corporate office's primary objective is to scrutinize the cost implications associated with different support base allocation and location strategies. This endeavor holds substantial significance within the corporate framework, as it is pivotal in formulating viable programming and budgeting plans. The assessment entails gauging performance at a specified level, considering localized and regional operations, their respective resource constraints (e.g., capacity, capabilities, etc.), and the objectives of individual local subsidiaries.

While each subsidiary is obligated to adhere to the policies dictated by the corporate office, they must simultaneously operate within the bounds of local regulations. Moreover, these entities must augment their operational prowess, which might be restricted by geographical, logistical, capacity-related, and transportation-based limitations. A notional geographical disposition of these twenty corporate subsidiaries and potential collaborators is itemized in the table below.

Our approach demands a meticulous assessment to ensure sufficient capacity is upheld to fulfill the requirements of the corporate strategic plans. At the local level, we must address tactical intricacies, including the costs associated with various support options and the timelines for their deployment, particularly under varying levels of stress experienced by local operations. This assessment considers factors such as infrastructure robustness, inherent characteristics of basing, logistical constraints, perturbations in the local environment, dynamically evolving requirements, and the gamut of day-to-day operational limitations such as the availability of a skilled labor pool.

Table 1: Notional Location of Subsidiaries and possible partners

Current Locations		**Potential Partner Locations**
Senegal	Greece	Qatar
Bahrain	India	Pakistan
Bulgaria	Italy	Singapore
Oman	Japan	Thailand
Germany	Nigeria	Ecuador
UK	Panama	Azerbaijan
Puerto Rico	Philippines	

This paper delineates how predicaments can be effectively modeled as bilevel optimization problems (see also Amouzegar and Moshirvaziri, 2011). Subsequently, we furnish a comprehensive suite of solutions tailored to the aforementioned specific scenario.

BILEVEL PROGRAMMING PROBLEM

The bilevel programming problem (BLPP) is a mathematical model of the leader-follower game. In this game, the control of decision variables is partitioned between the leader and the follower. The basic leader-follower strategy was initially proposed for a duopoly by von Stackelberg (1959), in which decisions are made sequentially and cooperation is not allowed. Perfect information is assumed because both players know the other's objective functions and allowable strategies.

The leader moves first by choosing a vector $x \in X \subset R^{n_1}$ to optimize her objective function

$F(x; y)$. The leader's choice of strategies affects both the follower's objective and decision space. The follower observes the leader's choice and reacts by selecting a vector $y \in Y \subset R^{n_2}$ that optimizes her objective function $f(x, y)$ for a given x. In doing so, the follower affects the leader's outcome.

In general, a bilevel programming problem can be formulated as follows:

$$\min_{x,y} F(x,y)$$

$$x \in X = \{x|G(x) \geq 0\}$$

Where y solves

$$\min_{z} f(x,z)$$

$$g(x,y) \geq 0,$$

$$z \in Y = \{y|H(y) \geq 0\}$$

Where G, H and g are vector valued functions of dimensions n_1, n_2 and n_3, respectively. F and f are real valued functions of appropriate dimensions (Bard, 1988). The sets of X and Y may represent upper and lower bounds on elements of the vectors x and y. Bilevel Programming has wide applicability in network design, transport system planning, management, and economics, particularly central economic planning. An extensive review of bilevel optimization from classical to evolutionary approaches, including several applications, appears in Sinha and Deb (2018). Another important prior review of bilevel programming and applications appears in Kalashnikov (2015).

Further Analysis of BLPP

Consider the bilevel programming problem, and for a moment, assume all the constraints and functions are linear (i.e., linear bilevel programming problem). For example, the problem may be written as follows:

Problem (P)

$$\min_{x,y} F(x,y) = c^T x + d^T y$$

Where y solves

$$\min_{y} f(y) = e^T y$$

$$(x,y) \in \Omega$$

Where $\Omega = \{(x,y)|Ax + By \leq b, x \geq 0, y \geq 0\}$. Now, let the lower-level part of the problem be denoted by $L(x)$: $\psi(x) = Min\ \{f(x,y)|(x,y) \in \Omega\}$. Therefore, the original BLPP can be written as,

Problem (Q)

$$\min_{x,y} F(x,y)$$

$$\psi(x) \geq f(x,y)$$

$$(x,y) \in \Omega$$

Given that $\psi(x)$ is a convex function, problem (P) is also known as (LRCP), Linear program with an additional Reverse Convex constraint Problem (Moshirvaziri and Amouzegar, 2002). Clearly, problems (P) and (Q) are equivalent. This transformation allows for the use of certain efficient algorithms to solve this class of optimization problems (see Drezner and Kalczynski, 2019) for both linear and nonlinear bilevel programming.

Proposition: If Ω is nonempty and compact (closed and bounded), and if Problem (P) is solvable, then an optimal solution is achieved at a vertex of the polyhedron Ω.

Proof: See Moshirvaziri and Amouzegar, 2004

DETAILS OF LOCATION AND ALLOCATION MODEL

The model above, a bilevel programming framework, facilitates the exploration of various hypothetical scenarios and evaluates the spectrum of solutions regarding resource costs, considering diverse levels of support and operational capabilities.

Our analytical methodology comprises a series of discrete steps:

- An initial selection of a comprehensive range of scenarios, projecting forthcoming industry demands at each location while introducing system and subsystem perturbations to stress the supply chain resilience.
- Factors such as potential supply, services, and demands, coupled with the capacity and capabilities inherent to each subsidiary, govern the requisites for transportation, operations, storage alternatives, and other pertinent considerations.
- The optimization model identifies optimal storage facility (warehouse) placements, emphasizing minimizing operational and transportation costs associated with the envisaged demand and supply chains.

It should be underscored that the operational interplay of certain subsidiaries or partners may have minimal influence on the functioning of other entities within the system. Depending on the strategic orientation of the central office, specific local nodes might not play a role in particular operations. For instance, a strategic initiative to bolster medical device capabilities might necessitate geographical constraints. To account for this, we adopt an extended time horizon that fulfills a range of possible demands for the overall system and its constituent subsystems, aligning with distinct strategic objectives. The model optimally allocates resources and commodities across storage sites, determining the type and number of vehicles required for efficient material movement to operational locales. Consequently, a resilient transportation and allocation network materializes, seamlessly connecting disparate entities.

The analysis leads to several alternative operating strategies. The table below shows three different outcomes. To find the best overall solution, a comprehensive portfolio analysis is necessary and will depend on the parameters set by the leader in order to optimize the system costs while considering the goals and needs of each regional facility. Each facility aims to meet its operational needs, including local operations, transportation, and capacity limits, while aligning with the headquarters' strategic plans.

The final solution involves allocating resources to specific locations, considering transportation, operation, and maintenance costs, as well as the potential for new facility construction. This approach, which combines current and potential storage sites, results in an 18 percent decrease in total costs, primarily due to lower transportation expenses. These savings are realized separately for both existing and new locations, each optimized for different deployment scenarios.

CONCLUDING REMARKS

In this paper, we have introduced a comprehensive analytical framework aimed at scrutinizing alternative strategies concerning the allocation and location of resources and support within distinct regional settings. This endeavor is guided by a nuanced understanding of the intricate layers of decision-making that exhibit both overt and covert interdependencies. This unique approach propelled us to embrace a novel optimization technique, leveraging a bilevel programming paradigm. This formulation offers an avenue for assessing the true cost efficacy of the overarching supply chain, aligned with the imperatives of central strategic planning while harmonizing the distinct needs of semi-autonomous regional nodes. By adopting this methodology, we can mirror operational reality more faithfully, thereby devising solutions that align with corporate objectives and attain heightened operational efficiency.

Table 2: Three sample results for location and allocation of resources and operations

Current State	Result 1	Result 2	Result 3
Bahrain	Italy	Pakistan	Bahrain
Central Facility1(CF1)	CF1	CF1	CF1
Central Facility2(CF2)	Azerbaijan	Ecuador	Nigeria
Germany	Greece	Philippines	Singapore
India	CF2	CF2	CF2
Italy	Bahrain	India	Senegal
Japan	Qatar	Panama	UK
Nigeria	Germany	Japan	Pakistan
Oman/Qatar	UK	Thailand	Greece
Panama	Philippines	Germany	Germany
Senegal	Oman	Singapore	Qatar
Singapore	Panama	UK	
UK	Bulgaria	Puerto Rico	Thailand

Furthermore, this approach fosters a heightened mutual awareness between each stratum of decision-making, thus paving the way for holistic optimization of the entire operational spectrum rather than isolated segments. By cultivating an environment where every layer comprehends the direct repercussions of their choices upon other layers, we unlock the potential for systemic optimization—encompassing the entirety of operations, transcending the optimization of isolated components.

However, it is essential to acknowledge that bilevel programming problems fall within the domain of NP-hard complexities, rendering the solution of large-scale instances a formidable challenge. It is commonplace for many modelers to seek solace in converting these problems into single-level, large-scale optimization paradigms utilizing conventional mathematical programming techniques. While this transformation could yield a range of policy alternatives, it often fails to capture the intricate interplay that characterizes decisions at distinct hierarchical levels.

It is imperative to recognize that, given the nuanced nature of the problem, an optimization-derived solution cannot serve as the sole determinant in the policy formulation process. While the bilevel model brings us closer to the actual complexities of the problem, its inherent complexity introduces challenges that must be acknowledged. A multi-faceted and well-informed approach, enriched by the integration of diverse perspectives, remains indispensable for effective policymaking in this context.

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