# A GEOMETRIC MEDIAN DECOMPOSITION METHOD TO MINIMIZE AGGREGATE FIT LOSS

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## ABSTRACT

Accurate bra sizing is crucial for effectiveness. This study presents a Geometric Median Decomposition (GMD) computational algorithm designed to minimize fitting errors, which is essential given the limitations of large-scale customization. Incorrect sizing can lead to discomfort and inadequate support.Custom-fitted bras notably outperform mass-produced ones. GMD's iterative refinement of the geometric median offers a potent solution for diverse body shapes, bridging the gap between size and fit. This method marks a significant stride in enhancing bra sizing techniques.

**Keywords:** Modeling, Simulation, Optimization, Fit Loss Model, Advanced Design and Manufacturing, Bra Fit.

## INTRODUCTION

The efficacy of a bra design hinges on the precision of its sizing. Even a well-engineered bra will only provide satisfactory support if its dimensions are accurate. This paper delves into the central concern of bra sizing, presenting a method to minimize overall fit loss. This solution is vital due to the inherent limitations of large-scale customization. While full customization allows for tailored products or services, offering personalized sizing options (Da Silveira, et al., 2001), the constraints of current 3D manufacturing and the imprecision in measurement and imaging techniques necessitate additional refinement through mathematical modeling.

Inaccurate sizing gives rise to a myriad of challenges, resulting in insufficient breast support, discomfort, and tissue protrusion. Achieving an appropriate bra fit is critical for women, particularly those who have undergone breast surgery, as it significantly impacts their physical and psychological well-being. Unfortunately, existing bra sizing paradigms inadequately capture the intricate nuances of breast contours and dimensions, often relying on basic measurements like under-bust and over-bust circumferences (Bowles et al., 2012). In addition, the lack of standardization in bra sizing and measurement methods across bra brands has further worsened the ability to find the right fit (McGhee & Steele, 2020).

Empirical research reveals that custom-fitted garments, including bras, offer superior fit and greater satisfaction compared to mass-produced alternatives (Lanier, 2020). Despite the potential benefits, personalized attire remains limited due to various factors such as time constraints, design intricacies, and labor-intensive processes (White & Scurr, 2012). Innovations such as 3D scanning, printing, and knitting methods have emerged as a promising solution. These innovative techniques have the potential to produce custom-fitted products on a larger scale, making personalized attire more accessible. However, as previously noted, there will always be a gap between the actual size and the fit, highlighting the need for methods to minimize this loss.

## EXISTING SIZING APPROACH

The alphabetical bra sizing system, introduced by Warner's company in 1935, has remained the same since its inception. This widely adopted system uses various methods to calculate band and cup sizes despite the ongoing scrutiny of measurement accuracy and reliability (Peterson & Suh, 2019). The band size is determined by adding five inches to the under-bust measurement in inches (for odd numbers) or four inches (for even numbers). The cup size is the difference between the over-bust measurement and the estimated band size. In contrast, the "metric system" directly derives the band size from the under-bust measurement. This system is primarily used in Europe and Asia and occasionally in the US. The cup size is calculated by subtracting the under-bust measurement from the over-bust measurement. In Australia and New Zealand, bra sizes are measured in centimeters, with the band size corresponding to one's "dress size," increasing in steps of two (e.g., 10, 12, and 14). However, the dress size does not align with the bra band size, leading to potential confusion. Each band size reflects a measured difference of 5 cm, and cup sizes are denoted by letters (e.g., C, D, DD, and E), representing a difference of 2 cm. For example, a size ten band corresponds to an under-bust measurement of 68-72 cm, while a size 12 corresponds to 73-77 cm. A 10A cup size corresponds to an over-the-bust measurement of 82-84 cm, while a 10B cup size is 84-86 cm.

### Sizing inequities

The limited and standardized range of bra sizes has led to inconsistent sizing standards and ill-fitting bras, failing to accommodate the unique body shapes of individuals. This issue is even more pronounced for post-surgery bras, which may require additional fitting and sizing needs compared to mainstream bras. In a recent survey (Amoozegar-Montero, et al., 2022), the majority of participants (92%) reported breast discomfort due to incorrect bra size and fit, including improper band measurement and ill-fitting cups (too tight or loose). Additional issues include the bra moving around or not sitting correctly on the body and bra straps, causing discomfort. Post-surgery discomforts, such as skin sensitivity, irritation from underwire, and scar aggravation, exacerbate the problem. Difficulty fastening the bra, temperature and humidity-related discomfort, and post-surgical side effects like fat necrosis, swelling, and chest wall pain further contribute to discomfort. Furthermore, poorly fitting bras cause physical discomfort and affect the wearer's internal and external perceptions. Individuals expect support, comfort, proper fit, and a bra that stays in place (for more insights into the sociological aspects of bras and bra wearers, refer to Amoozegar-Montero, 2022).

However, despite various attempts by researchers and manufacturers, achieving the perfect bra fit for each individual remains a complex challenge due to women's bodies' unique and ever-changing nature. Several studies (Amoozegar-Montero, 2024; Wang & Suh, 2019; Gorea et al., 2020) have explored innovative design and process options, such as 3D scanning, 3D printing and 3D knitting systems for improved bra solutions. Nevertheless, until accurate individual scanning is feasible (with each change in body shape and structure) and a personalized manufacturing process becomes available for every bra need, we must work within the confines of a discrete sizing process. This approach offers more options and accurate sizing translation, though it still falls short. To advance bra design and development, we propose a mathematical solution based on the concept of aggregate fit loss and a methodology for its optimization.

## AGGREGATE-FIT-LOSS

A loss function, also known as a cost function or error function, is a mathematical function that takes the values of one or more variables or events and maps them to an actual number, intuitively indicating a measure of the loss related to a given event or values (see Hastie, et al., 2001). A fit-loss function evaluates how well a garment conforms to the size of an individual within a specific sizing system, measuring the degree of fit suitability (see McCulloch, et al., 1998). "Aggregate Fit Loss" (AFL) denotes the cumulative gauge of the fitting error or inconsistency between a dataset of data points and a model or benchmark point (see also Pei, et al., 2020). It encompasses a collective evaluation of individual data points, quantifying the overall precision or quality of the model's adherence to the data.

Consider a set of n data points (e.g., a 3D scan of an individual), denoted as $\{x_{1,}, x_2, \dots, x_m\}$ and let y be the reference point or model (e.g., a baseline sizing).

The aggregated fit loss is typically computed as the sum of individual fit losses between each data point and the model.

$$AFL = \sum_i \left|\left|x_i - y\right|\right|_2 \qquad (1)$$

Where, $||x_i - y||_2$ is called norm2, and in details, it is calculated by the following expression,

$$||x_i - y||_2 = \sqrt{\sum_{j=1}^{m}(x_j^{(i)} - y_j)^2},$$

In this notation, $x_j^{(i)}$ denotes the j-th component of vector $x_i$ for $, j = 1, \dots, m$.

A smaller aggregated fit loss value indicates a better overall fit of the model to the data, meaning that the model is closer to the majority of data points. Conversely, a higher aggregated fit loss suggests a poorer fit, indicating that the model deviates more from the data points.

The concept of aggregated fit loss is often used in optimization problems, where the goal is to find the model or reference point that minimizes the aggregated fit loss, ensuring the best fit to the data. For example, in the context of bra fitting, minimizing the aggregated fit loss helps to find the geometric median that best fits the 3D body scans and improves comfort and support in bras for individuals with diverse body shapes and sizes.

## Geometric Median

The geometric median, also known as the Fermat-Weber point or 1-median, is a point that minimizes the sum of Euclidean distances to a set of points in Euclidean space. In essence, envision a multitude of points scattered across a map, where the objective is to pinpoint a specific location that optimizes the cumulative distance from that location to all other points (see Figure 1). This location is termed the geometric median. In the realm of bra fitting, the geometric median assumes the role of identifying the optimal fit for an array of bras, accommodating distinct individuals.

In a more concrete sense, consider a set of bras, each assessed by numerous individuals encompassing diverse body shapes and sizes. The geometric median is akin to discovering the most suitable fit for the bra size and shape that minimizes overall dissatisfaction across the gamut of individuals who have tried on the bras.

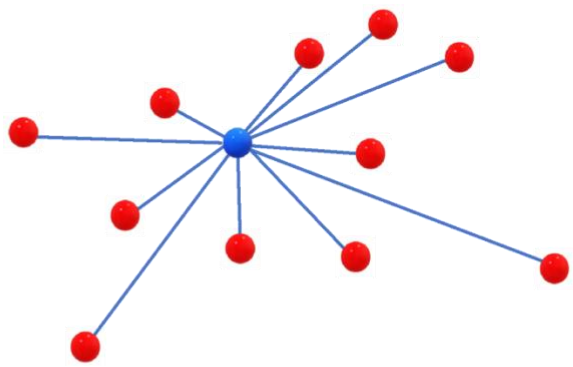

Figure 1: Example of Geometric Median. The blue dot represents GMD, and the red circles are the scanned data

Consequently, the identification of the geometric median is indispensable. By discerning the central fit that satisfies a majority of individuals, it becomes plausible to fashion bras that are inclined to offer enhanced comfort and support across a diverse user base. Mathematically akin to the previously mentioned fit-loss function, the minimization of fit-loss materializes as Geometric Median Decomposition (GMD).

Mathematically, for a given asset of m points, $\{x_{1,}, x_2, \dots, x_m\}$, where $x_i \in R^n$, then the geometric median is defined as

$$\underset{y \in R^n}{\arg min} \sum_{j=1}^{m} || x_j - y || \tag{2}$$

Despite its intuitive essence, the geometric median presents a considerable computational challenge, particularly within the domain of bra fitting. While the centroid, effectively minimizing the summation of squared distances to each point, can be readily computed using a straightforward formula involving average point coordinates, the geometric median lacks a comparably direct solution. Nevertheless, several methods exist, including Weiszfeld's algorithm and Geometric Median Decomposition (GMD).

The Weiszfeld method is an iterative re-weighted least square and has the following form:

$$Z_{K+1} = \frac{\sum_{k=1}^{n}\left(\frac{x_k}{||x_k - Z_K||}\right)}{\sum_{k=1}^{n}\left(\frac{1}{||x_k - Z_K||}\right)} \quad (3)$$

**Geometric Median Decomposition (GMD)**

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However, in the context of this research, powerful iterative numerical techniques that converge toward an accurate estimate of the geometric median can be employed. These iterative methods iteratively refine the estimate, approaching the actual geometric median with each iteration until a satisfactory solution is reached.

The Geometric Median Decomposition (GMD) algorithm (See Jiang, 2005) is used to find the best-fitting point among a group of scattered points. In this case, we are interested in finding an ideal point in the middle of all the given points that may have been derived by scanning dozens of people.

Formally, the algorithm is as follows:

Given a set of $n$ points, $\{x_1, x_2, \cdots, x_n\}$, the geometric median, denoted as GMD, is the point $m$ that minimizes the function:

$$F(m) = \sum_{i=1}^{n} \| x_i - m \| \quad (4)$$

Initialize the geometric median estimate GMD_0 as the centroid of the initial scan or randomly select a point from the scan as the initial estimate.

For each iteration k from 1 to K:

> Consider the k-th scan and calculate the distances from each point in the scan to the current estimate GMD_(k-1).
>
> Assign weights to each point in the scan based on their distances to GMD_(k-1). Points closer to GMD_(k-1) receive higher weights
>
> Compute the weighted geometric median for the $k^{th}$ scan, which is the point that minimizes the weighted sum of Euclidean distances.
>
> Set GMD_k as the weighted geometric median obtained above
>
> Repeat for K iterations or until convergence criteria are met

The GMD algorithm offers several advantages for aggregating scans and finding the geometric median:

**Iterative Refinement:** By dividing the problem into multiple subproblems and refining the estimate iteratively, the GMD algorithm converges to a more accurate geometric median, providing a robust and reliable solution.

**Handling Outliers**: GMD can effectively handle outliers in the scans. Points far from the initial estimate, GMD_0, will have lower weights, reducing their influence on the final estimate.

**Flexibility:** The GMD algorithm is adaptable to different scan distributions and dimensionalities, making it suitable for a wide range of bra fitting scenarios.

**Improved Fit Loss Minimization:** By optimizing the aggregate fit loss across multiple scans, the GMD algorithm aims to provide bras that offer better comfort, support, and fit for individuals with diverse body shapes and sizes.

The GMD algorithm is a powerful tool for minimizing aggregate fit loss and improving bra fitting outcomes. Its ability to iteratively refine estimates and handle outliers contributes to a more accurate and personalized fitting experience, addressing the longstanding challenges of traditional bra fitting methods.

### GMD Methodology

To implement the GMD algorithm for improved bra fitting, a dataset of 3D body scans from individuals with diverse body shapes and sizes is needed. This dataset encompasses a range of breast volumes, chest circumferences, and torso shapes to ensure that the resulting bra designs cater to the needs of a diverse population.

Before the GMD algorithm application, the acquired 3D scans necessitate preprocessing to ensure uniformity and compatibility. This preprocessing step entails aligning scans to a common reference frame and normalizing data to eliminate scaling or rotation disparities. Once alignment and normalization are achieved, the GMD algorithm is brought to bear to locate the geometric median corresponding to the optimum bra fit, reducing aggregate fit loss across all scans.

The iterative nature of the GMD algorithm allows for fine-tuning the geometric median estimate at each step, considering the weights assigned to each point based on its distance from the previous estimate. The algorithm continues refining the estimate until it converges the specified number of iterations (K).

Although we are interested in the Aggregate Fit Loss, other loss metrics can be used to evaluate the performance of the GMD-based bra fitting approach. These metrics should capture different aspects of bra fit, including comfort, support, and individual fit loss for each scan.

Common fit loss metrics include:

**Aggregate Fit Loss:** The sum of individual fit losses for all scans, quantifying the overall fit performance of the GMD-based fitting approach.

**Individual Fit Loss:** The fit loss of each scan is calculated as the Euclidean distance between the scanned points and the estimated geometric median.

**Comfort and Support Ratings:** Participants can provide subjective feedback on the comfort and support provided by the bras resulting from the GMD algorithm. This feedback helps assess the practicality of the algorithms' outputs in real-world scenarios.

### An Illustrative Example

For the simplicity of the illustration, assume we are interested in developing a single size bra that would have the best fit for 100 women of a similar body type. The process includes under and over and other measures (e.g., the distance between the breasts, etc.). The underbust measurements range from 72.1cm to 76.5cm (e.g., a 34-sized bra). The cup size ranged from 82 to 86. This is equivalent to a 34C in commercial bra stores (see Figure 2)

The bases for a generic bra for under- and overbust are 74 and 90 cm, respectively, providing an aggregated fit loss of 172.95. However, using the algorithm above, we can set a new baseline (GMD) of 74.44 and 89.23, which has an aggregate fit loss of 150, which provides a 13.1% increase in better fit across the population that may consider itself a 34C-sized bra wearer.

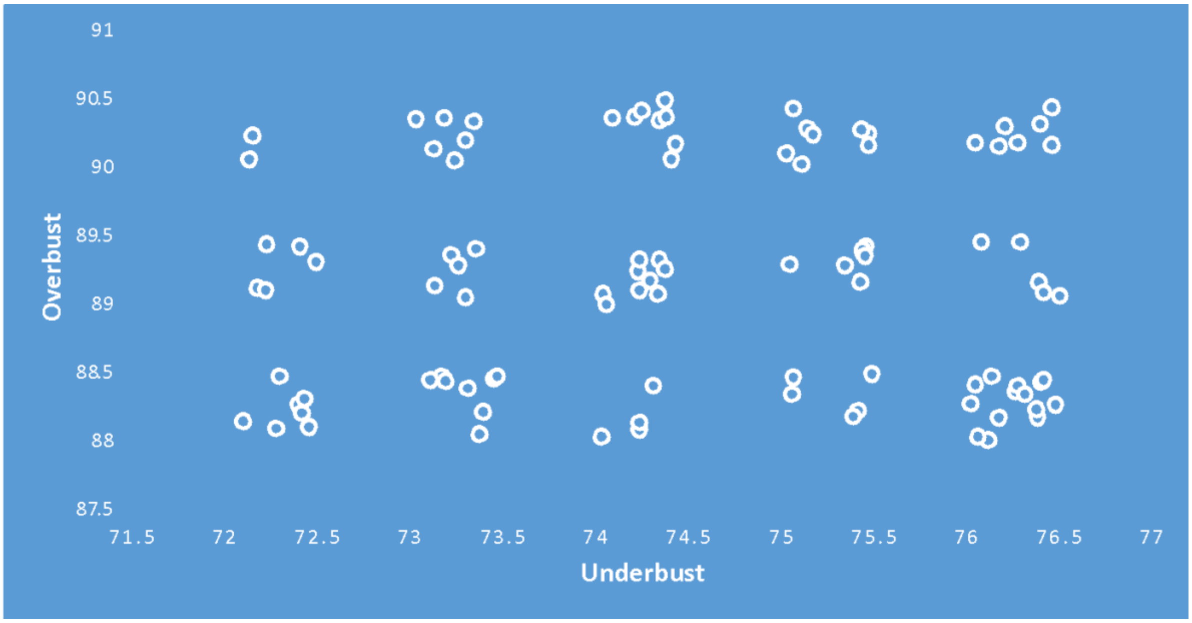


*Figure 2: Range of underbust and overbust size*

Considering the cup volume (base and heights, though not its accurate contour), we can add two sets of data and minimize the fit loss model. The result is the following solution for our sample with about 12.91% improvement.

**Table 1**: Results based on cup volume

| | **Original** | **Optimized** |
|---|---|---|
| **Underbust** | 74.00 | 74.48 |
| **Overbust** | 90.00 | 89.23 |
| **Cup Diameter** | 11.40 | 11.38 |
| **Cup Height** | 5.72 | 5.72 |

Given a relatively significant difference between bust and cup size and possible independence between the point measurement, we ran four 1-D modes with the following results:

**Table 2**: Results of 1-D mode simulation

| | Original | Optimized |
|---|---|---|
| **Underbust** | 74.00 | 74.37 |
| **Overbust** | 90.00 | 89.17 |
| **Cup Diameter** | 11.40 | 11.39 |
| **Cup Height** | 5.72 | 5.69 |

This resulted in an aggregate fit loss improvement of 12.7%, which is not as good as the first run of the model. Either variation of the options (e.g., using volume or normalizing the data) did not provide a better solution.

**Direct Optimization Method**.

To verify our results obtained via the Weiszfeld method, we alternatively consider solving problem (2) of minimization of AFL to find the optimal value of fit loss and, in particular, the solution vector y (the Geometric Median) by applying optimization via CVX/MATLAB. This package specifies and solves convex programs (See Grant and Boyd, 2013). It is important to note that problem (2) is a special form of an important class of optimization problems called Second Order Cone Programming (SOCP). This later class of programs is known to be in the *Convex Optimization* domain.

This fact enables us to code and implement problem (2) in higher dimensional space with confidence in the solution. Additionally, the accuracy of our computational results in the MATLAB environment is known to be reliable. This is especially true when the output parameters are the results of iterative techniques, where minor deviation in the accuracy of a variable in one iteration could propagate quickly and enlarge in the later iterations.

**Verification of prior results**

For our test-data discussed in the previous section and tested with the Weiszfeld method, we implemented and solved our optimization problem (2) using CVX/MATLAB.

The results of the 1-D model are consistent with those obtained via the Weiszfeld method and are presented below.

**Table 3**: Consistency of 1-D model and the Weiszfeld method

| | Original | Optimized |
|---|---|---|
| **Underbust** | 74.00 | 74.477508254528402 |
| **Overbust** | 90.00 | 89.230426802972843 |
| **Cup Diameter** | 11.40 | 11.390806336118080 |
| **Cup Height** | 5.72 | 5.7135703946596530 |

This resulted in an aggregate fit loss improvement of 12.91%, slightly better but consistent with that obtained via the Weiszfeld method. The same trend/pattern applies to the solution of various scenarios tested with the earlier method, thus verifying the superiority of using the Geometric Median Decomposition (GMD) algorithm to optimize the Aggregate Fit-Loss function (AFL).

## CONCLUDING REMARKS

This paper addresses the critical issue of precise bra sizing and fit by proposing a method to minimize overall fit loss, which is vital due to limitations in large-scale customization. While existing bra sizing paradigms have struggled to capture the diverse contours and dimensions of the human body, the proposed mathematical solution in this paper introduces a promising avenue for improvement.

This paper introduces the concept of Aggregate Fit Loss (AFL) and the Geometric Median Decomposition (GMD) algorithm to bridge the gap between actual size and fit. AFL quantifies fitting discrepancies between empirical data and predictive models to minimize overall fit loss. GMD algorithm identifies a geometric median that optimally fits various body shapes. The iterative GMD process effectively refines the estimate, catering to individual differences and handling outliers, thus enhancing the overall fitting experience.

The paper underscores the necessity for innovative bra sizing approaches, demonstrating the GMD algorithm's potential to mitigate fit loss and offer more accurate and personalized fitting experiences. The application of these concepts holds promise for addressing longstanding challenges in traditional bra sizing methodologies and improving comfort, support, and fit for individuals with diverse body shapes and sizes. Through an interdisciplinary lens, this paper presents a concrete methodology for translating mathematical insights into tangible improvements in bra design.

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